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ROXY INDEX: AN INDICATIVE INSTRUMENT TO MEASURE THE SPEED OF SPATIAL CONCENTRATION AND DECONCENTRATION OF POPULATION

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CONTENTS

ABSTRACT

The present paper addresses a question on how to measure the speed of changes in the spatial distribution pattern of population. Particularly discussed are some characteristics of the ROXY index which the author has developed as a comprehensive index to determine the degree of acceleration or deceleration of the speed of population concentration or deconcentration in a system of spatial units.

To put it concretely, the ROXY index (Type II) for the period between time t and time $t+1$ that is the revised version of the ROXY index (Type I), is defined as follows;

$$\text{ROXY index}=\left\{\frac{\text{growth ratio (weighted average)}}{\text{growth ratio (simple average)}}-1.0\right\}\times 10000$$

where growth ratio (weighted average)

$$=\sum_{i=1}^{n} X_{i,t+1}/\sum_{i=1}^{n} X_{i,t}$$

growth ratio (simple average)

$$=\sum_{i=1}^{n} (X_{i,t+1}/X_{i,t})\times\frac{1}{n}$$

$X_{i,\tau}$: Population level of spatial unit i at time τ
n : Number of spatial units

In the above definition, the spatial units could be, for example, census tracts, cities, metropolitan areas, rural areas or regions.

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This paper also presents the results of an empirical analysis in which the ROXY index was employed as an instrument to investigate the speed of population changes in the urban system of Japan as compared with that of the U.S. during the period 1960–80. The results suggest (i) that, for an urban system of Japan which is composed of her largest thirty metropolitan areas, the spatial deconcentration of population started in the first half of the 1970s and it will be continuously accelerated toward the 1990s and (ii) that, for an urban system of the U.S. which is composed of her largest thirty metropolitan areas, the speed of spatial deconcentration of population has been decelerated since the 1960s.

1. INTRODUCTION

There are a number of measurements to determine the distribution pattern of a specific attribute over a system of its constituent units or clusters. In the field of income distribution analysis, Lorenz (19) proposed in 1905 a new method to measure the inequality of wealth. He developed in his work a special curve representing the relationships between "cumulative population ratio against the total population" and "corresponding cumulative income ratio against the total income." This curve which is now famous as Lorenz curve, is one of the basic classical instruments to be used for investigating not only the pattern of income distribution but also the concentration or deconcentration phenomena in general. Among those who proposed other types of measurements for the inequality of income distribution are Gibra (11), Gini (12), Theil (22) and Atkinson (1).

Approximately half a century after Lorenz, Rodgers (20, 21) having benefited from the concept of Lorenz curve developed the idea of diversification index to summarize the extent to which a given area's industrial structure is similar to the national average industrial structure. Besides Rodgers' diversification index, various measurements of concentration or deconcentration have been constructed and investigated especially since 1940's for different kinds of industrial structure analysis. Enumerating some of them, we have coefficient of scatter defined by Creamer (4) as the least number of areal units necessary to account for 75 percent of total employment for a given industry, coefficient of geographic association defined by Florence et al. (10) on relative share of employment between two given industries, factor of redistribution suggested by Zelinsky (23) as defined on changes in per capita value added of a given industry, index of concentration defined by Herfindahl (14) and Hirschman (15) on market shares of industrial output, urbanization coefficient proposed by Duncan (6) on retail sales by area-size classes, index of dissimilarity defined by Duncan and Duncan (7) on employees' share by area between two occupational groups, index of segregation defined by Duncan and Duncan (8) on employees' share of a given occupation against total employees by area, concentration indices studied by Davies (5) for the inequality in the distribution of sales share by firm, and the measurement of concentration considered by Blackorby et al. (3) in their industrial performance analysis.

Hoover (16) was, on the other hand, interested in the spatial distribution of population and developed the coefficient of concentration defined on population share by area. This measurement is nowadays widely known as Hoover index. He proposed, in addition, the coefficient of redistribution to measure the difference in population share by area at different points of time. Bachi (2) who was also interested in the spatial distribution pattern of population, proposed the coefficient of population dispersion which indicates the magnitude of the spread of population independent of areal deleneation.

For this sphere of population redistribution dynamics, as a matter of fact, the present paper attempts to propose a plain comprehensive index, tentatively named ROXY index, in order to determine the direction, speed and speed-change of the spatial redistribution population. We now proceed, without any further remarks on other existing measurements[1)] which can be used for determining the degree of concentration or deconcentration of attributes, to Section 2 in which some fundamental characteristics of the proposed index are discussed.

2. ROXY INDEX: CONSTRUCTION

To begin, suppose we consider a system of three urbanized regions, say A, B and C. Suppose further that Regions A, B and C have population of 50, 250 and 500 persons respectively at time T_0 totalling 800 as shown in Table 1. Assume that Region A which is the smallest among the three regions in terms of the size of population increases its population by 10% during the period between time T_0 and T_1 to have the population of 55 at T_1. Also assume that the population level of Region A remains the same during the period between T_1 and T_2 followed by the decrease in population by 40% during the period between T_2 and T_3 to have the population of 33 at time T_3. In Region C which is the largest in population size, the growth rates of population are granted to be 30%, 100% and 150% for the periods of T_0 through T_1, T_1 through T_2 and T_2 through T_3 respectively, which results in the population of 650 at T_1, 1,300 at T_2 and 3,250 at T_3. As to Region B which is middle in population size among the three regions, its population changes at the growth rates higher than Region A but lower than Region C throughout the entire period between T_0 and T_3, viz. at the growth rates of 20%, 50% and 80% for the periods of T_0 through T_1, T_1 through T_2, and T_2 through T_3 respectively. As a result, Region B has the population of 300 at T_1, 450 at T_2 and 810 at T_3.

Looking at the change in the percentage share of population by region against total population of three regions which shifts from 800 at T_0 to 1,005 at T_1, 1,805 at T_2 and 4,093 at T_3, one can see that Region C continuously expands its population share from 62.50% at T_0 to 64.68% at T_1, 72.02% at T_2 and 79.40 at T_3 while Region A's share drastically contracts from 6.25% at T_0, to 5.47% at T_1, 3.05% at T_2 and 0.81% at T_3. The population share of Region B shows a steadily decreasing tendency changing from 31.25% at T_0, to 29.85% at T_1, 24.93% at T_2 and 19.79% at T_3.

For this hypothetical urban system, it might perhaps be acceptable to say in a broad

Table 1 ROXY Index: Accelerating Concentration of Population

Population percent-share of a larger spatial unit continues to increase as time goes on, while that of a smaller spatial unit decreases. The discrepancy over population growth rates among those spatial units tends to diverge.

(unit of population: person)

SPATIAL UNIT \ TIME	T_0		T_1		T_2		T_3
A	50 [6.25]		55 [5.47]		55 [3.05]		33 [0.81]
(GROWTH RATE)		(10%)		(0%)		(−40%)	
B	250 [31.25]		300 [29.85]		450 [24.93]		810 [19.79]
(GROWTH RATE)		(20%)		(50%)		(80%)	
C	500 [62.50]		650 [64.68]		1300 [72.02]		3250 [79.40]
(GROWTH RATE)		(30%)		(100%)		(150%)	
ALL UNITS	800 [100.00]		1005 [100.00]		1805 [100.00]		4093 [100.00]
(GROWTH RATE)		(25.63%)		(79.60%)		(126.76%)	

PERIOD	$T_0 \sim T_1$	$T_1 \sim T_2$	$T_2 \sim T_3$
WEIGHTED AVERAGE GROWTH RATIO (X)	1.2563	1.7960	2.2676
SIMPLE AVERAGE GROWTH RATIO (Y)	1.2000	1.5000	1.6333
ROXY INDEX (X/Y)	1.0469	1.1973	1.3884

(Note) 1. The figure in brackets [ ] indicates percent-share of total population for each spatial unit.

2. The figure in parentheses ( ) shown between columns for time T_i and time T_{i+1} indicates population growth rate of each spatial unit for the period between T_i and T_{i+1}.

way that the population tends to concentrate as time goes on because Region C's population share continues to increase while Region A's share decreases and that the speed of population concentration is accelerating because the growth rate of population in Region C continuously increases while in Region A the growth rate continuously decreases.

In Table 2, we have another hypothetical framework in which the population tends to concentrate until time T_2 followed by the balanced growth during the period between T_2 and T_3. However, the speed of concentration is decelerating throughout the period from T_0 to T_2 because the growth rate of population Region C continuously decreases while in Region A the growth rate continuously increases.

Tables 3 and 4 show additional two hypothetical frameworks. The former shows the case for population deconcentration with accelerating speed, while the latter is for the case of population deconcentration with decelerating speed[2].

As with the above setting, we might seek simple measures of dynamic processes of spatial population redistribution. One such measure would be the ROXY index as defined in Table 5. The computation of the value of the index for a specific time period be-

Table 2 ROXY Index: Decelerating Concentration of Population

Population percent-share of a larger spatial unit continues to increase until time T_2, while that of a smaller spatial unit decreases. The discrepancy over population growth rates among those spatial units, however, tends to converge as time goes on.

(unit of population: person)

SPATIAL UNIT \ TIME	T_0		T_1		T_2		T_3
A	50 [6.25]		50 [3.70]		55 [2.87]		66 [2.87]
(GROWTH RATE)		(0%)		(10%)		(20%)	
B	250 [31.25]		300 [22.22]		360 [18.80]		432 [18.80]
(GROWTH RATE)		(20%)		(20%)		(20%)	
C	500 [62.50]		1000 [74.08]		1500 [78.33]		1800 [78.33]
(GROWTH RATE)		(100%)		(50%)		(20%)	
ALL UNITS	800 [100.00]		1350 [100.00]		1915 [100.00]		2298 [100.00]
(GROWTH RATE)		(68.75%)		(41.85%)		(20%)	

PERIOD	$T_0 \sim T_0$	$T_1 \sim T_2$	$T_2 \sim T_3$
WEIGHTED AVERAGE GROWTH RATIO (X)	1.6875	1.4185	1.2000
SIMPLE AVERAGE GROWTH RATIO (Y)	1.4000	1.2667	1.2000
ROXY INDEX (X/Y)	1.2054	1.1198	1.0000

(Note) 1. The figure in brackets [ ] indicates percent-share of total population for each spatial unit.

2. The figure in parentheses ( ) shown between colums for time T_i and time T_{i+1} indicates population growth rate of each spatial unit for the period between T_i and T_{i+1}.

3. Balanced growth of population takes place during the period between T_2 and T_3.

Table 3 ROXY Index: Accelerating Deconcentration of Population

Population percent-share of a larger spatial unit continues to decrease as time goes on, while that of a smaller spatial unit increases. The discrepancy over population growth rates among those spatial units tends to diverge.

(unit of population: person)

SPATIAL UNIT \ TIME	T_0		T_1		T_2		T_3
A	50 [6.25]		65 [7.10]		130 [11.51]		325 [22.18]
(GROWTH RATE)		(30%)		(100%)		(150%)	
B	250 [31.25]		300 [32.79]		450 [39.82]		810 [55.29]
(GROWTH RATE)		(20%)		(50%)		(80%)	
C	500 [62.50]		550 [60.11]		550 [48.67]		330 [22.53]
(GROWTH RATE)		(10%)		(0%)		(−40%)	
ALL UNITS	800 [100.00]		915 [100.00]		1130 [100.00]		1465 [100.00]
(GROWTH RATE)		(14.37%)		(23.50%)		(29.65%)	

Table 3 (continued)

PERIOD	$T_0 \sim T_1$	$T_1 \sim T_2$	$T_2 \sim T_3$
WEIGHTED AVERAGE GROWTH RATIO (X)	1.1437	1.2350	1.2965
SIMPLE AVERAGE GROWTH RATIO (Y)	1.2000	1.5000	1.6333
ROXY INDEX (X/Y)	0.9531	0.8233	0.7938

(Note) 1. The figure in brackets [ ] indicates percent-share of total population for each spatial unit.
2. The figure in parentheses ( ) shown between columns for time T_i and time T_{i+1} indicates population growth rate of each spatial unit for the period between T_i and T_{i+1}.

Table 4 ROXY Index: Decelerating Deconcentration of Population

Population percent-share of a larger spatial unit continues to decrease until time T_2, while that of a smaller spatial unit increases. The discrepancy over population growth rates among those spatial units, however, tends to converge as time goes on.

(unit of population: person)

TIME / SPATIAL UNIT	T_0		T_1		T_2		T_3
A	50 [6.25]		100 [11.11]		150 [14.15]		180 [14.15]
(GROWTH RATE)		(100%)		(50%)		(20%)	
B	250 [31.25]		300 [33.33]		360 [33.96]		432 [33.96]
(GROWTH RATE)		(20%)		(20%)		(20%)	
C	500 [62.50]		500 [55.56]		550 [51.89]		660 [51.89]
(GROWTH RATE)		(0%)		(10%)		(20%)	
ALL UNITS	800 [100.00]		900 [100.00]		1060 [100.00]		1272 [100.00]
(GROWTH RATE)		(12.50%)		(17.78%)		(20%)	

PERIOD	$T_0 \sim T_1$	$T_1 \sim T_2$	$T_2 \sim T_3$
WEIGHTED AVERAGE GROWTH RATIO (X)	1.1250	1.1778	1.2000
SIMPLE AVERAGE GROWTH RATIO (Y)	1.4000	1.2667	1.2000
ROXY INDEX (X/Y)	0.8036	0.9298	1.0000

(Note) 1. The figure in brackets [ ] indicates percent-share of population for each spatial unit.
2. The figure in parentheses ( ) shown between columns for time T_i and time T_{i+1} indicates population growth rate of each spatial unit for the period between T_i and T_{i+1}.
3. Balanced growth of population takes place during the period between T_2 and T_3.

tween t and $t+1$ requires the following steps. First, calculate the weighted average growth rate (WAGRate) of population by equation (1) in Table 5. For this calculation, the population by each spatial unit is used as weighting factor. Therefore, WAGRate is necessarily equal to the growth rate of total population. Second, calculate the simple average growth

Table 5 Growth Rate, Growth Ratio and ROXY Index (For the period between time t and $t+1$)

1. Weighted average growth rate (WAGRate) in terms of %

$$\sum_{i=1}^{n}\left[\frac{X_{i,t}}{\sum_{j=1}^{n}X_{j,t}}\left\{\left(\frac{X_{i,t+1}}{X_{i,t}}-1.0\right)\times 100\right\}\right]$$

$$=100\frac{1}{\sum_{i=1}^{n}X_{i,t}}\sum_{i=1}^{n}\left(\frac{X_{i,t}X_{i,t+1}}{X_{i,t}}-X_{i,t}\right)$$

$$=100\left(\frac{\sum_{i=1}^{n}X_{i,t+1}}{\sum_{i=1}^{n}X_{i,t}}-1.0\right) \quad \cdots\cdots(1)$$

2. Simple average growth rate (SAGRate) in terms of %

$$\sum_{i=1}^{n}\left[\frac{1}{n}\left\{\left(\frac{X_{i,t+1}}{X_{i,t}}-1.0\right)\times 100\right\}\right]$$

$$=\frac{100}{n}\sum_{i=1}^{n}\left(\frac{X_{i,t+1}}{X_{i,t}}-1.0\right) \quad \cdots\cdots(2)$$

3. Weighted average growth ratio (WAGRatio)

$$1.0+\frac{\text{WAGRate}}{100}$$

$$=\frac{\sum_{i=1}^{n}X_{i,t+1}}{\sum_{i=1}^{n}X_{i,t}} \quad \cdots\cdots(3)$$

4. Simple average growth ratio (SAGRatio)

$$1.0+\frac{\text{SAGRate}}{100}$$

$$=\frac{1}{n}\sum_{i=1}^{n}\frac{X_{i,t+1}}{X_{i,t}} \quad \cdots\cdots(4)$$

5. ROXY index

$$\frac{\text{WAGRatio}}{\text{SAGRatio}}$$

$$=\frac{\sum_{i=1}^{n}X_{i,t+1}}{\sum_{i=1}^{n}X_{i,t}}\times\frac{n}{\sum_{i=1}^{n}\frac{X_{i,t+1}}{X_{i,t}}}$$

$$=\frac{\sum_{i=1}^{n}(r_i^{t,t+1}\times X_{i,t})}{\sum_{i=1}^{n}X_{i,t}}\times\frac{n}{\sum_{i=1}^{n}r_i^{t,t+1}} \quad \cdots\cdots(5)$$

(Note) 1. ROXY stands for <u>R</u>atio <u>O</u>f "Weighted Average Growth Ratio (abbreviated as <u>X</u>)" to "Simple Average Growth Ratio (abbreviated as <u>Y</u>)."

2. $X_{i,\tau}$: Population level of spatial unit i at time τ, $X_{i,\tau}>0$ (for $\tau=t,\ t+1$).

2. n: Number of spatial units.

4. $r_i^{t,t+1}$: Growth ratio of population in spatial unit i for the period between time t and time $t+1$.

rate (SAGRate) by equation (2). Third, calculate the weighted average growth ratio (WAG-Ratio) from WAGRate through equation (3). Fourthly, calculate the simple average growth ratio (SAGRatio) from SAGRate through equation (4). Finally, calculate the value of ROXY index as the ratio between WAGRatio and SAGRatio by equation (5)[3].

The value of ROXY index calculated through the above steps is always positive. If all the spatial units grow or decline at the same growth or decline rate for a given period of time, the population shares of each regions remain fixed resulting in that the value of ROXY index turns out to be unity. If the population is concentrating, then the value is expected to be greater than unity[4]. In this case, higher speed of concentration seems to correspond to larger value of the index. In contrast, if the population is deconcentrating, then the value of the index is expected to fall into the range between zero and unity[5]. In this case, the higher speed of deconcentration seems to correspond to smaller value of the index.

Besides these basic features of the value of ROXY index, a time series of ROXY indices could also provide the information on the changes in the speed of population concentration or deconcentration. More precisely, if the population concentration is accelerating, unchanging or decelerating over successive periods of time, then the value of the index is expected to increase, remain constant or decrease respectively and to be always greater than unity. On the contrary, if the population deconcentration is accelerating, unchanging or decelerating, then the value of the index is expected to decrease, remain constant or increase respectively and to be always between zero and unity.

Based on the above observation, one can construct Table 6 to show (i) the relationship between the value of ROXY index and spatial redistribution pattern, and (ii) that between the direction of change in the value of ROXY index and change in the speed of spatial redistribution. In this table two kinds of ROXY index, Type I and Type II, are shown. The former is the same ROXY index as already defined in Table 5, while the latter is the revised version of ROXY index obtained through the following equation:

ROXY Index of Type II

$=(\text{ROXY Index of Type I}-1.0)\times 10^4$.

The ROXY index of Type II has been developed for the practical purpose of (i) making the index to have positive value for the case of concentration and negative value for the case of deconcentration and (ii) enabling us to more easily perceive the difference between two values of the index.

The relationships between columns (a) through (d) in Table 6 seem to work rather well for the four numerical examples described in Tables 1 through 4. In Table 1, the ROXY index of Type I has the values of 1.0469, 1.1973 and 1.3884 for the periods of T_0 through T_1, T_1 through T_2 and T_2 through T_3 respectively. For this example where the value of ROXY index is always greater than unity and increases as time goes on, Table 6 indicates that the spatial distribution of population is acceleratingly concentrating. The value of the index changes from 1.2054 to 1.1198 and to 1.0000 in Table 2. For this example, Table 6 indicates that the spatial distribution of population is deceleratingly concentrat-

Table 6 What Dose the ROXY Index Tell Us?

	For Measuring the Direction and Speed of Spatial Redistribution		For Measuring the Change in the Speed of Spatial Redistribution	
	(a)	(b)	(c)	(d)
	Value of ROXY	Spatial Redistribution Pattern of Population Share	Change in the Value of ROXY	Change in the Speed of Spatial Concentration or Deconcentration
ROXY INDEX (Type I)	ROXY>1.0	Concentration	↗ (1) → (2) ↘ (3)	(1) Accelerating (2) Constant (3) Decelerating
	ROXY=1.0	Stable Share	1.0→1.0	Balanced Growth or Balanced Decline
	0<ROXY<1.0	Deconcentration	↗ (1) → (2) ↘ (3)	(1) Decelerating (2) Constant (3) Accelerating
ROXY INDEX (Type II)	ROXY>0	Concentration	↗ (1) → (2) ↘ (3)	(1) Accelerating (2) Constant (3) Decelerating
	ROXY=0	Stable Share	0.0→0.0	Balanced Growth or Balanced Decline
	ROXY<0	Deconcentration	↗ (1) → (2) ↘ (3)	(1) Decelerating (2) Constant (3) Accelerating

ing between T_0 and T_2 and that the balanced growth takes place between T_2 and T_3. In Table 3, the index shows the values of 0.9531, 0.8233 and 0.7938. For this example where the value of ROXY index is always less than unity and decreases as time goes on, Table 6 indicates that the spatial distribution of population is acceleratingly deconcentrating. The value changes from 0.8036, to 0.9298 and to 1.0000 in Table 4. For this example, Table 6 indicates that the spatial distribution is deceleratingly deconcentrating between T_0 and T_2 and that the balanced growth takes place between T_2 and T_3.

It should be however clearly kept in mind that the contents expressed in a column (b) of Table 6 seem to compose sufficient conditions for those in its corresponding column (a), but the other way round does not necessarily hold[6]. Though, except when we are dealing with very special situations, we may presumably not have to worry too much about the possible lack of one-to-one correspondence between the conditions in column (a) and those in column (b).

3. ROXY INDEX: APPLICATION

To help demonstrate a plausible way for the application of ROXY index in empirical analyses, let us calculate the values of ROXY index for the data on metropolitan population in both Japan and the U.S. In Japan, there exist eighty-six mertopolitan areas defined as Functional Urban Cores (FUCs). The information on the population levels in the five consecutive census years from 1960 through 1980 for all FUCs and for central cities of the largest thirty FUCs, is provided in Table 7. From this table, we can construct Table 8 showing the population growth rates for the largest thirty FUCs and their central cities. In the U.S., there were 323 metropolitan areas on June 30, 1981, defined as Standard Metropolitan Statistical Areas (SMSAs). Out of them, the largest thirty SMSAs have been chosen into Table 9 which shows changes in population of the thirty SMSAs and their central cities for the period 1940–80[7].

Based on Tables 8 and 9, we get Table 10 showing the values of ROXY index (Type II)[8] for (i) an urban system of Japan which is composed of the largest thirty FUCs for four consecutive five-year periods from 1960 through 1980 and (ii) an urban system of the U.S. which is composed of the largest thirty SMSAs for the periods of 1960–70, 70–75 and 75–80. For the urban system of Japan, one can see from this table that the value of ROXY index for the thirty FUCs continuously decreases from 89.2 for the period 1960–65 to −19.3 for the period 1975–80 during which the positive sign of the index changes to negative around the year 1970. This would imply that before 1970 the population growth rates of "larger" FUCs were in general higher than those of "less-larger" FUCs[9], bur that the discrepancy in population growth rates between "larger" FUCs and "less-larger" FUCs was becoming narrower until 1970. For a while around 1970, the general balanced growth of population took place as to the thirty FUCs. A slightly after 1970, however, the population growth rates of "larger" FUCs turned out to be in general lower than those of "less-larger" FUCs. Since then the discrepancy in population growth rates between "larger" FUCs and "less-larger" FUCs has been becoming wider with "less-larger" FUCs showing higher growth rates than "larger" FUCs. In other words, the spatial distribution of population in the urban system of the thirty FUCs was deceleratingly concentrating until around 1970. After that the concentration of population ceased and the deconcentration started. This tendency of population deconcentration was continuously accelerated throughout the 1970s[10].

For the U.S. urban system, the sign of ROXY index remains nagative in the entire period 1960–80, but the absolute value of the index gradually decreases. This would imply that the population growth rates (or decline rates) of "larger" SMSAs have always been in general lower (higher) than those of "less-larger" SMSAs since 1960, but that the discrepancy in population growth rates or decline rates between "larger" SMSAs and "less-larger" SMSAs has been becoming gradually narrower. In other words, the spatial distribution of population in the urban system of the thirty SMSAs was deceleratingly deconcentrating in a continuous manner since 1960[11].

Therefore, if we would assume that the spatial dynamics of Japanese urban system would more or less follow the historical path of the urban system of the U.S. which is regarded as "the most advanced country with respect to urbanization," then we might be

Table 7 Population Changes in Japan (1960–1980): For Functional Urban Cores (FUCs)

FUC and CC	Rank (1980 FUC Population)	Population					Nr. of localities
		1960	1965	1970	1975	1980	
Sapporo	7	887,535	1,101,329	1,310,693	1,558,739	1,745,345	5
CC	—	615,628	821,217	1,010,123	1,240,617	1,401,758	—
Hakodate	50	322,970	331,804	343,406	362,637	380,514	5
Asahikawa	55	239,636	271,930	297,189	320,526	352,620	1
Muroran	71	201,221	227,200	238,137	242,941	241,428	3
Kushiro	70	178,731	198,984	214,922	231,403	242,331	3
Obihiro	73	159,846	175,329	189,643	203,004	221,662	4
Aomori	61	253,952	264,921	279,294	303,055	327,298	3
Hirosaki	67	232,842	229,993	231,520	237,813	248,963	6
Hachinohe	62	253,474	264,767	281,838	297,473	312,343	7
Morioka	48	286,736	301,530	318,532	348,174	382,814	8
Sendai	10	860,509	922,607	1,019,991	1,160,920	1,271,318	21
CC	—	425,272	480,925	545,065	615,473	664,799	—
Ishinomaki	76	188,427	187,376	191,066	197,905	204,465	6
Akita	42	401,513	404,280	415,990	438,920	466,697	13
Yamagata	45	383,092	382,153	391,335	409,933	435,632	7
Fukushima	51	319,768	325,801	338,403	358,500	376,944	8
Aizuwakamatsu	82	175,162	171,115	167,605	168,710	174,616	6
Kouriyama	49	309,223	316,187	332,688	356,581	381,819	4
Mito	31	411,235	430,161	462,343	509,530	550,432	12
Hitachi	53	318,134	331,419	335,157	348,301	360,799	6
Utsunomiya	21	564,682	583,921	625,795	697,120	752,827	14
CC	—	239,007	265,696	301,231	344,417	377,748	—
Maebashi	54	279,557	297,136	318,747	341,323	360,252	6
Takasaki	43	353,262	368,552	391,387	424,747	451,370	10
Kiryu	80	159,393	164,427	171,730	179,798	183,934	4
Chiba	12	540,852	642,330	838,299	1,077,675	1,224,611	9
CC	—	258,357	339,850	482,133	659,344	746,428	—
Tokyo	1	13,388,959	15,844,973	18,005,894	19,955,814	21,049,507	121
CC	—	8,310,027	8,893,094	8,840,942	8,642,800	8,349,209	—
Yokohama	4	2,272,380	2,901,289	3,603,704	4,258,008	4,592,642	15
CC	—	1,375,510	1,788,915	2,238,264	2,621,648	2,773,822	—
Odawara	63	233,572	263,399	283,736	302,690	311,927	9
Niigata	18	657,650	684,250	713,690	762,831	815,390	14
CC	—	325,018	356,302	383,919	423,204	457,783	—
Nagaoka	69	212,790	218,177	224,121	233,008	242,976	4
Toyama	32	477,794	480,192	493,522	522,486	547,056	11
Takaoka	47	367,534	363,314	364,085	376,284	384,157	8
Kanazawa	24	482,871	507,897	540,268	600,819	647,139	13
CC	—	313,112	335,828	361,379	395,262	417,681	—
Fukui	33	485,114	493,737	499,568	526,470	546,360	15
Koufu	44	382,963	385,021	398,003	421,891	443,777	16
Nagano	40	404,489	413,282	429,191	460,582	484,568	11
Matsumoto	57	288,435	293,499	306,225	326,626	346,645	10
Gifu	13	805,117	886,222	959,945	1,043,477	1,103,051	23
CC	—	312,597	358,259	385,727	408,699	410,368	—
Shizuoka	14	793,848	860,971	927,563	993,432	1,031,374	8

Table 7 (Continued)

FUC and CC	Rank (1980 FUC Population)	Population					Nr. of localities
		1960	1965	1970	1975	1980	
CC	—	350,897	382,799	416,378	446,952	458,342	—
Hamamatsu	15	743,710	779,062	827,403	891,775	945,941	17
CC	—	357,098	392,632	432,221	468,886	490,827	—
Numazu	37	330,878	374,863	421,513	468,590	495,140	7
Fuji	58	244,499	265,534	294,619	326,039	340,703	4
Nagoya	3	3,642,667	4,201,059	4,714,576	5,180,943	5,430,025	64
CC	—	1,697,093	1,935,430	2,036,053	2,079,694	2,087,884	—
Toyohashi	30	403,935	439,617	473,409	520,769	554,283	8
CC	—	215,515	238,672	258,547	284,597	304,274	—
Toyota	29	311,142	364,410	445,073	525,850	590,135	5
CC	—	104,529	136,728	197,193	248,774	281,609	—
Tsu	52	310,101	317,047	329,540	351,405	367,414	10
Ise	79	174,001	177,547	178,606	183,663	186,481	7
Otsu	38	302,222	322,270	356,159	424,452	488,437	8
Kyoto	5	1,511,077	1,644,808	1,809,412	1,984,788	2,085,076	15
CC	—	1,284,818	1,365,007	1,419,165	1,461,050	1,472,993	—
Osaka	2	6,855,068	8,298,236	9,521,577	10,374,705	10,694,672	68
CC	—	3,011,563	3,156,222	2,980,487	2,778,975	2,648,158	—
Kobe	6	1,441,703	1,588,300	1,740,999	1,908,784	1,988,253	8
CC	—	1,113,977	1,216,640	1,288,937	1,360,530	1,367,392	—
Himeji	16	682,238	732,534	782,646	838,691	871,119	18
CC	—	334,520	373,653	408,353	436,099	446,255	—
Nara	46	209,160	238,931	289,195	352,723	404,259	5
Wakayama	28	491,841	534,381	572,343	601,362	617,128	11
CC	—	285,155	328,657	365,267	389,677	401,462	—
Tottori	75	204,752	200,044	199,035	204,715	213,535	11
Yonago	74	189,769	189,817	192,831	203,758	216,709	10
Matsue	68	226,178	224,096	227,877	236,758	248,093	9
Okayama	20	583,686	605,213	647,614	719,828	765,680	15
CC*	—	306,757	338,693	375,106	513,452	545,737	—
Kurashiki	36	337,115	355,369	418,465	480,215	497,686	9
Hiroshima	11	732,365	861,374	994,560	1,166,010	1,258,864	12
CC*	—	431,336	504,245	541,998	852,607	899,394	—
Kure	60	321,224	329,580	335,273	342,540	337,427	10
Fukuyama	26	475,869	491,050	544,938	604,910	622,780	7
CC*	—	183,682	204,768	255,086	329,779	346,031	—
Shimonoseki	59	331,874	332,023	328,801	336,848	340,391	5
Ube	72	242,216	220,085	211,317	221,869	229,752	4
Yamaguchi	85	136,097	130,218	130,685	135,517	145,066	3
Iwakuni	81	168,067	175,221	174,427	181,402	182,936	5
Tokushima	35	447,679	449,893	458,585	484,487	510,425	13
Takamatsu	22	594,749	595,973	617,272	667,985	705,740	21
CC	—	243,538	257,716	274,367	298,997	316,662	—
Matsuyama	34	389,653	413,531	445,917	499,017	542,284	8
Imabari	78	176,467	176,809	181,583	192,296	197,397	7
Niihama	77	197,286	194,550	193,238	200,679	203,468	3
Kochi	41	367,439	383,774	405,169	443,577	470,870	9

Table 7 (Continued)

FUC and CC	Rank (1980 FUC Population)	Population					Nr. of localities
		1960	1965	1970	1975	1980	
Kitakyushu	9	1,518,451	1,515,708	1,501,563	1,554,303	1,604,577	19
CC	—	986,401	1,042,388	1,042,321	1,058,067	1,065,084	—
Fukuoka	8	1,089,452	1,197,739	1,348,113	1,565,142	1,744,420	24
CC*	—	661,395	749,808	853,270	1,002,214	1,088,617	—
Omuta	65	345,890	325,751	297,188	290,578	290,772	6
Kurume	39	462,451	452,729	456,193	466,017	487,704	15
Saga	64	295,715	286,643	283,571	289,675	304,956	11
Nagasaki	27	506,565	523,700	545,435	592,092	617,302	8
CC*	—	380,983	405,479	421,114	450,195	447,091	—
Sasebo	66	297,099	273,533	272,294	275,668	277,479	3
Kumamoto	19	625,931	643,565	671,565	718,481	783,397	16
CC*	—	373,922	407,052	440,020	488,053	525,613	—
Yatsushiro	86	152,094	145,623	140,809	140,019	143,279	4
Oita	25	474,068	491,972	520,798	587,009	630,798	10
CC	—	207,151	226,417	260,584	320,236	360,484	—
Miyazaki	56	247,866	257,218	274,925	310,210	349,620	6
Miyakonojyo	84	148,052	143,481	138,538	142,667	155,712	3
Nobeoka	83	148,223	147,559	151,337	157,639	161,216	3
Kagoshima	23	490,734	515,900	543,018	601,595	663,069	11
CC	—	334,643	371,129	403,340	456,818	505,077	—
Naha	17	555,764	619,847	666,131	767,619	828,563	21
CC	—	223,047	257,177	276,380	295,091	295,801	—
All FUCs	—	60,670,350	67,639,667	74,731,360	82,275,810	86,988,636	1,024

(Note)

1. FUC stands for functional urban core which is defined as Japanese-version of SMSA by T. Kawashima and N. J. Glickman. See T. Kawashima (17) for the details of the definition of FUC.
2. Figure for population is as of October 1.
3. FUC boundaries are as of 1970 and fixed over the time.
4. CC stands for central city. The population of the central city is given for each of the largest thirty FUCs. The boundaries of central cities are as of 1980 and fixed over the time. For central city with * mark, population in 1960, 1965 and 1970 is given for the 1970 boundary of that city, and population in 1975 and 1980 is given for the 1980 boundary of that city.
5. Eighty-six FUCs covers 8,596,511 ha which is 23% of national territory. The total population residing in these FUCs as a fraction of the national total population was 74.31% in 1980.
6. The number of localities composing each FUC is as of October, 1970.
7. Total population of the largest thirty FUCs was 44,985,418 (1960), 51,580,237 (1965), 58,034,287 (1970), 64,481,476 (1975), and 68,235,026 (1980).

Table 8 Population Growth Rates of the Largest Thirty Functional Urban Cores (FUCs) and Their Central Cities (1960–1980)

FUC	Rank (1980 FUC Population)	Spatial Unit	Population Growth Rate					Rank among 30 FUC's	
			1960–65	1965–70	1970–75	1975–80	1975–80 (annual)	1975–80 PGR	1980 Pop.
Tokyo	1	FUC	18.3	13.6	10.8	5.5	1.07	5	1
		CC	7.0	−0.6	−2.2	−3.4	−0.69	—	—
Osaka	2	FUC	21.1	14.7	9.0	3.1	0.61	3	2
		CC	4.8	−5.6	−6.8	−4.7	−0.96	—	—
Nagoya	3	FUC	15.3	12.2	9.9	4.8	0.94	8	3
		CC	14.0	5.2	2.1	0.4	0.08	—	—
Yokohama	4	FUC	27.7	24.2	18.2	7.9	1.52	1	4
		CC	30.1	25.1	17.1	5.8	1.13	—	—
Kyoto	5	FUC	8.9	10.0	9.7	5.1	0.99	13	5
		CC	6.2	4.0	3.0	0.8	0.16	—	—
Kobe	6	FUC	10.2	9.6	9.6	4.2	0.82	10	6
		CC	9.2	5.9	5.6	0.5	0.10	—	—
Sapporo	7	FUC	24.1	19.0	18.9	12.0	2.29	2	8
		CC	33.4	23.0	22.8	13.0	2.47	—	—
Fukuoka	8	FUC	9.9	12.6	16.1	11.5	2.19	12	7
		CC*	13.4	13.8	17.5	8.6	1.67	—	—
Kitakyushu	9	FUC	−0.2	−0.9	3.5	3.2	0.64	30	9
		CC	5.7	−0.0	1.5	0.7	0.13	—	—
Sendai	10	FUC	7.2	10.6	13.8	9.5	1.83	18	11
		CC	13.1	13.3	12.9	8.0	1.55	—	—
Hiroshima	11	FUC	17.6	15.5	17.2	8.0	1.54	6	10
		CC*	16.9	7.5	57.3	5.5	1.07	—	—
Chiba	12	FUC	18.8	30.5	28.6	13.6	2.59	4	12
		CC	31.5	41.9	36.8	13.2	2.51	—	—
Gifu	13	FUC	10.1	8.3	8.7	5.7	1.12	11	13
		CC	14.6	7.7	6.0	0.4	0.08	—	—
Shizuoka	14	FUC	8.5	7.7	7.1	3.8	0.75	16	14
		CC	9.1	8.8	7.3	2.5	0.50	—	—
Hamamatsu	15	FUC	4.8	6.2	7.8	6.1	1.19	21	15
		CC	10.0	10.1	8.5	4.7	0.92	—	—
Himeji	16	FUC	7.4	6.8	7.2	3.9	0.76	17	16
		CC	11.7	9.3	6.8	2.3	0.46	—	—
Naha	17	FUC	11.5	7.5	15.2	7.9	1.54	9	17
		CC	15.3	7.5	6.8	0.2	0.05	—	—
Niigata	18	FUC	4.0	4.3	6.9	6.9	1.34	22	18
		CC	9.6	7.8	10.2	8.2	1.58	—	—
Kumamoto	19	FUC	2.8	4.4	7.0	9.0	1.75	28	19
		CC*	8.9	8.1	10.9	7.7	1.49	—	—
Okayama	20	FUC	3.7	7.0	11.2	6.4	1.24	24	20
		CC	10.4	10.8	36.9	6.3	1.23	—	—

Table 8 (Continued)

FUC	Rank (1980 FUC Population)	Spatial Unit	Population Growth Rate					Rank among 30 FUC's	
			1960–65	1965–65	1970–70	1975–75	1975–80 (annual)	1975–80 PGR	1980 Pop.
Utsunomiya	21	FUC	3.4	7.2	11.4	8.0	1.55	25	21
		CC	11.2	13.4	14.3	9.7	1.86	—	—
Takamatsu	22	FUC	0.2	3.6	8.2	5.7	1.11	29	22
		CC	5.8	6.5	9.0	5.9	1.15	—	—
Kagoshima	23	FUC	5.1	5.3	10.8	10.2	1.96	20	24
		CC	10.9	8.7	13.3	10.6	2.02	—	—
Kanazawa	24	FUC	5.2	6.4	11.2	7.7	1.50	19	26
		CC	7.3	7.6	9.4	5.7	1.11	—	—
Oita	25	FUC	3.8	5.9	12.7	7.5	1.45	23	28
		CC	9.3	15.1	22.9	12.6	2.40	—	—
Fukuyama	26	FUC	3.2	11.0	11.0	3.0	0.58	27	23
		CC*	11.5	24.6	29.3	4.9	0.97	—	—
Nagasaki	27	FUC	3.4	4.2	8.6	4.3	0.84	26	27
		CC*	6.4	3.9	6.9	−0.7	−0.14	—	—
Wakayama	28	FUC	8.6	7.1	5.1	2.6	0.52	15	25
		CC	15.3	11.1	6.7	3.0	0.60	—	—
Toyota	29	FUC	17.1	22.1	18.1	12.2	2.33	7	29
		CC	30.8	44.2	26.2	13.2	2.51	—	—
Toyohashi	30	FUC	8.8	7.7	10.0	6.4	1.26	14	30
		CC	10.7	8.3	10.1	6.9	1.35	—	—
Average (Weighted)		FUC	14.7	12.5	11.1	5.8	1.14		
		CC**	10.5	5.1	4.2	1.3	0.25		
Average (Simple)		FUC	9.7	10.1	11.4	6.8	1.33		
		CC**	13.6	11.6	10.4	5.0	0.98		
Japan			5.2	5.5	7.0	4.6	0.90		

(Note)
1. CC Stands for central city.
2. See note 4 of Table 7 for the central cities with * mark.
3. **: Excluding Fukuoka, Hiroshima, Kumamoto, Okayama, Fukuyama and Nagasaki cities.

Table 9 Population Changes in the US (1940–1980): For the Largest Thirty SMSAs and Their Central Cities

SMSA	Rank (1980 SMSA Pop.)	Spatial Unit	Population (1,000)				Population Growth Rate (%)				(Reference) PGR (%)	
			1960	1970	1975	1980	1960–70	1970–75	1975–80	1970–80	1940–50	1950–60
New York	1	SMSA	9,540	9,974	9,561	9,120	4.5	−4.1	−4.6	−8.6	—	A
		CC	7,782	7,896	7,482	7,072	1.5	−5.2	−5.5	−10.4	5.9	−1.4
Los Angeles-	2	SMSA	6,039	7,042	6,987	7,478	16.6	−0.8	7.0	6.2	—	45.5
Long Beach		CC	2,479	2,812	2,727	2,967	13.4	−3.0	8.8	5.5	31.0	25.8
Chicago	3	SMSA	6,221	6,977	7,015	7,104	12.2	0.5	1.3	1.8	—	20.1
		CC	3,550	3,369	3,099	3,005	−5.1	−8.0	−3.0	−10.8	6.6	−2.0
Philadelphia	4	SMSA	4,343	4,824	4,807	4,717	11.1	−0.4	−1.9	−2.2	—	18.3
		CC	2,003	1,949	1,816	1,688	−2.7	−6.8	−7.0	−13.4	7.3	−3.3
Detroit	5	SMSA	3.950	4,435	4,424	4,353	12.3	−0.2	−1.6	−1.8	—	A
		CC	1,670	1,514	1,335	1,203	−9.3	−11.8	−9.9	−20.5	14.0	−9.7
San Francisco-	6	SMSA	2,649	3,109	3,140	3,251	17.4	1.0	3.5	4.6	—	24.0
Oakland		CC	740	716	665	679	−3.2	−7.1	2.1	−5.2	20.1	−4.5
Washington	7	SMSA	2,097	2,910	3,022	3,061	38.8	3.8	1.3	5.2	—	A
		CC	764	757	712	638	−0.9	−5.9	−10.4	−15.7	21.0	−4.7
Dallsa-	8	SMSA	1,738	2,378	2,527	2,975	36.8	6.3	17.7	25.1	—	A
Ft. Worth		CC	680	844	813	904	24.1	−3.7	11.2	7.1	47.1	56.7
Houston	9	SMSA	1,430	1,999	2,286	2,905	39.8	14.4	27.1	45.3	—	A
		CC	938	1,234	1,357	1,595	31.6	10.0	17.5	29.3	54.8	57.4
Boston	10	SMSA	2,688	2,899	2,890	2,763	7.8	−0.3	−4.4	−4.7	—	A
		CC	697	641	637	563	−8.0	−0.6	−11.6	−12.2	3.9	−13.0
Nassau-	11	SMSA	1,967	2,556	2,657	2,606	29.9	4.0	−1.9	2.0	—	—
Suffolk		CC	(NA)	(NA)	(NA)	(NA)	—	—	—	—	—	—
St. Louis	12	SMSA	2,144	2,411	2,367	2,356	12.5	−1.8	−0.5	−2.3	—	A
		CC	750	622	525	453	−17.1	−15.6	−13.7	−27.2	5.0	−12.5
Pittsburgh	13	SMSA	2,405	2,401	2,322	2,264	−0.2	−3.3	−2.5	−5.7	—	8.7
		CC	604	520	459	424	−13.9	−11.7	−7.6	−18.5	0.7	−10.8
Baltimore	14	SMSA	1,804	2,071	2,148	2,174	14.8	3.7	1.2	5.0	—	A
		CC	939	905	852	787	−3.6	−5.9	−7.6	−13.0	10.6	−1.2
Minneapolis-	15	SMSA	1,598	1,965	2,011	2,114	23.0	2.3	5.1	7.6	—	A
St. Paul		CC	434	434	378	371	−10.1	−12.9	−1.9	−14.5	6.1	−7.5
Atlanta	16	SMSA	1,169	1,596	1,790	2,030	36.5	12.2	13.4	27.2	—	A
		CC	487	495	436	425	1.6	−11.9	−2.5	−14.1	9.6	47.1
Newark	17	SMSA	1,833	2,057	1,999	1,966	12.2	−2.8	−1.7	−4.4	—	A
		CC	405	382	340	329	−5.7	−11.0	−3.2	−13.9	2.1	−7.7
Anaheim-	18	SMSA	704	1,421	1,700	1,933	101.8	19.6	13.7	36.0	—	225.6
Santa Ana-		CC	104	166	194	219	59.6	16.9	12.9	31.9	36.4	593.3
Garden Grove												
Cleveland	19	SMSA	1,909	2,064	1,967	1,899	8.1	−4.7	−3.5	−8.0	—	A
		CC	876	751	639	574	−14.3	−14.9	−10.2	−23.6	4.2	−4.3
San Diego	20	SMSA	1,033	1,358	1,585	1,862	31.5	16.7	17.5	37.1	—	85.5
		CC	573	697	774	876	21.6	11.0	13.2	25.7	64.5	71.6
Maiami	21	SMSA	935	1,268	1,439	1,626	35.6	13.5	13.0	28.2	—	88.9
		CC	292	335	365	347	14.7	9.0	−4.9	3.6	44.8	17.3
Denver-	22	SMSA	935	1,239	1,413	1,621	32.5	14.0	14.7	30.8	—	A
Boulder		CC	494	515	485	492	4.3	−5.8	1.4	−4.5	29.2	18.8

Table 9 (Continued)

SMSA	Rank (1980 SMSA Pop.)	Spatial Unit	Population (1,000) 1960	1970	1975	1980	Population Growth Rate (%) 1960–70	1970–75	1975–80	1970–80	(Reference) PGR (%) 1940–50	1950–60
Seattle-Everett	23	SMSA	1,107	1,425	1,407	1,607	28.7	−1.3	14.2	12.8	—	31.1
		CC	557	531	487	494	−4.7	−8.3	1.4	−7.0	27.2	19.0
Tampa-St. Petersburg	24	SMSA	809	1,089	1,348	1,569	34.6	23.8	16.4	44.1	—	A
		CC	275	278	280	272	1.1	0.7	−2.9	−2.2	15.7	120.0
Riverside-San Bernadino-Ontario	25	SMSA	810	1,141	1,226	1,558	40.9	7.4	27.1	36.5	—	A
		CC	84	140	151	171	66.7	7.9	13.2	22.1	34.3	78.7
Phoenix	26	SMSA	664	969	1,221	1,509	45.9	26.0	23.6	55.7	—	100.0
		CC	439	582	665	790	32.6	14.3	18.8	35.7	64.6	310.3
Cincinnati	27	SMSA	1,268	1,385	1,381	1,401	9.2	−0.3	1.4	1.2	—	24.0
		CC	503	453	413	385	−9.2	−8.8	−6.8	−15.0	10.5	−0.2
Milwaukee	28	SMSA	1,279	1,404	1,409	1.397	9.8	0.4	−0.9	−0.5	—	A
		CC	741	717	666	636	−3.2	−7.1	−4.5	−11.3	8.5	16.3
Kansas City	29	SMSA	1,109	1,274	1,290	1,327	14.9	1.3	2.9	4.2	—	A
		CC	476	507	473	448	6.5	−6.7	−5.3	−11.6	14.5	4.2
San Jose	30	SMSA	642	1,065	1,174	1,295	65.9	10.2	10.3	21.6	—	A
		CC	204	460	556	629	125.5	20.9	13.1	36.7	39.7	214.7
Total		SMSA	66,819	78,706	80,513	83,841	17.8	2.3	4.1	6.5		
		CC*	30,589	31,222	29,781	29,436	2.1	−4.6	−1.2	−5.7		
Average (Weighted)		SMSA	2,227	2,524	2,684	2,795	17.8	2.3	4.1	6.5		
		CC*	1,055	1,077	1,027	1,015	2.1	−4.6	−1.2	−5.7		
Average (Simple)		SMSA	2,227	2,524	2,684	—	26.2	5.4	7.0	13.3		
		CC*	1,055	1,077	1,027	—	10.1	−2.8	−0.2	−2.3		
United States			179,323	203,302	215,465	226,546	13.37	5.98	5.14	11.4		

* Excluding the city of Nassau

(Note)

1. The figure for population is as of April 1.
2. SMSA boundaries are as of 1980 and fixed over the time.
3. CC stands for central city. In case there are more than one central cities for an SMSA, CC presents the city with the largest 1980 population among them.
4. Boundaries of the central cities are not fixed but variable over the time.
5. PGR stands for population growth rate. Due to the limits of available data, population growth rates of SMSAs for the period 1950–60 are shown only for those SMSAs whose boundaries remained fixed between 1966 and 1980. For other SMSAs, we put symbol A if the population growth rate for the period 1950–1960 is positive for the 1966 SMSA boundary. For the SMSAs which were not existing in 1966, we put symbol A in case we can reasonably gather that the growth rates of central cities of those SMSAs are positive for the period 1950–60.
6. The percentage share of the total population of the largest thirty SMSAs against the national total population is: 37.26% for 1960, 38.71% for 1970, 37.37% for 1975 and 37.01% for 1980.
7. NA means "not available."

(Sources) US Bureau of the Census (1965, pp. 17–20; 1966, pp. 17–21; 1972, pp. 21–23; 1977, pp. 19–24; 1980, pp. 12, 21–26; 1981, pp. 18–23).

Table 10 ROXY Index (Type II) for Urbanization in Japan and the U.S.

(a) For Japan

Period / Group of Spatial Units	1960–65	1965–70	1970–75	1975–80
30 FUCs	89.2	42.8	−6.1	−19.3

(b) For the U.S.

Period / Group of Spatial Units	1960–70	1970–75	1975–80
30 SMSAs	−68.5	−59.0	−53.5

(Note) 1. ROXY Index (Type II) = {ROXY Index (Type I) − 1.0} × 10,000.
2. The values of ROXY index shown in this table have been calculated on the basis of the annual growth rates instead of the five-year growth rates or the ten-year growth rates.

able to say by comparing the figures in Tables 10 (a) and 10 (b) that the value of ROXY index for the largest thirty FUCs would possibly be getting continuously smaller until it reaches the value in the range between −50 and −100 and that the value would then gradually start to increase. In other words, the figures in Table 10 would imply that some of "larger" FUCs in Japan would most probably start losing their population in the foreseeable future though this interpretation should be considered only as a highly tentative one because many other types of empirical analyses have to be conducted to gain more accurate insight into the future growth or decline of the "larger" metropolitan areas in Japan.

4. CONCLUSION

The motivation of the small endeavor attempted in the present paper was just a subjective and naive expectation that the ratio between weighted average and simple average of population growth rates might be of some help in evaluating the degree of temporal changes in the spatial redistribution pattern of urban population. Partly because of such naiveness and subjectiveness and partly because of a quite sketchy approach by which the exercise in this paper has been undertaken, it should be admitted that the features of ROXY index is yet to be thoroughly investigated especially as to its mathematical implications and general applicability.

In spite of that, it would still be interesting to further explore potential usefulness of the index. For example, when we carry out the intra-metropolitan analysis on the pattern of spatial redistribution of population over subareas in a specific metropolitan area, the distance to the central business district from each subareas may be employed as weighting factor in the calculation of the value of ROXY index[12]. Among other possible weighting factors could be population densisty, transportation accessibility and structure-related-values[13].

On the same time, the ROXY index approach may be possibly useful for certain types of empirical studies on the spatial redistribution processes of not only population but also other variables such as employment, income, investment, production, transactions and consumption.

NOTES

1) Among the most important other existing measurements are, for example, those developed by Hart (13) and Finkelstein and Friedberg (9) to describe the concentration or deconcentration phenomena based on the concepts of entropy theory.

2) Note that in Table 4 the balanced growth takes place during the period of T_2 through T_3.

3) At the very initial stage in developing the ROXY index, the author provisionally applied the ratio between WAGRate and SAGRate to the index. This kind of index, however, has the following drawbacks.

(i) The value of the index would drastically change even if the SAGRate varies even very slightly in the vicinity of zero.

(ii) We should be faced with the sign problem in the sense that the identical message will come out for both cases of a/b and $-a/-b$ where a (or $-a$) and b (or $-b$) are the values of WAGRate and SAGRate respectively. The same thing can be pointed out for the cases of $a/-b$ and $-a/b$.

In order to avoid those two problems, the author transformed the "growth rate" into "growth ratio" through equations (3) and (4) in Table 5 to define the ROXY index (more strictly speaking, the ROXY index of Type I) as the ratio between WAGRatio and SAGRatio. Note that both of the weighted and simple average growth ratios between time t and time $t+1$ are always positive as long as the population levels at those two time points remain positive. Meanwhile, we can of course independently define the WAGRatio and SAGRatio without touching upon the WAGRate and SAGRate formulated in equations (1) and (2). Nevertheless, for the purpose of explaining the basic relationships between the concept of growth rate and that of growth ratio, WAGRatio and SAGRatio are expressed by means of the transformation from WAGRate and SAGRate in the definitional equations (3) and (4) respectively.

4) This proposition for the case of population concentration holds not only for the situation in which the total population is increasing but also for the situation in which the total population is decreasing.

5) This proposition for the case of population deconcentration holds not only for the situation in which the total population is increasing but also for the situation in which the total population is decreasing.

6) For instance, a careful investigation of equation (5) in Table 5 would tell us that if all spatial units have the identical level of population at time t then the value of ROXY index (Type I) turns out to be unity no matter how differently the population levels of each spatial units change during the period between time t and time $t+1$. One of such cases is numerically illustrated by Table N-1. In order to avoid this sort of problem, one could possibly develop an index defined as follows;

the arithmetic average of (i) the value of ROXY index for the period between time t and time $t+1$ which shall be calculated by employing the population level of each spatial unit at time t as its weighting factor and (ii) the value of ROXY index for the period between time t and time $t+1$ which shall be calculated by employing the population level of each spatial unit at time $t+1$ as its weighting factor.

Table N-2 shows, in mathematical formula, this arithmetic average as well as the weighted average growth ratio which can be obtained by use of the population level at time $t+1$ as

Table N-1 Example of the Case where All Spatial Units have the Same Level of Population at the Beginning of a Period

(unit of population: person)

SPATIAL UNIT \ TIME	T_0		T_1
A	10 [33.33]		10 [16.67]
(GROWTH RATE)		(0%)	
B	10 [33.33]		20 [33.33]
(GROWTH RATE)		(100%)	
C	10 [33.33]		30 [50.00]
(GROWTH RATE)		(200%)	
ALL UNITS	30 [100.00]		60 [100.00]
(GROWTH RATE)		(100%)	

PERIOD	$T_0 \sim T_1$
WEIGHTED AVERAGE GROWTH RATIO (X)	2.0000
SIMPLE AVERAGE GROWTH RATIO (Y)	2.0000
ROXY INDEX (Type I) (X/Y)	1.0000

(Note) 1 The figure in brackets [ ] indicates percent-share of total population for each spatial unit.
2 The figure in parentheses ( ) shown between columns for time T_t and time T_{t+1} indicates population growth rate of each spatial unit for the period between T_t and T_{t+1}.

Table N-2 Possible ROXY Index: Average of Two Values of ROXY Index with Different Weighting Factors

1. Weighted average growth ratio calculated by employing population level at time $t+1$ as weighting factor

$$\sum_{i=1}^{n}\left(\frac{X_{i,t+1}}{\sum_{j=1}^{n} X_{j,t+1}} \times \frac{X_{i,t+1}}{X_{i,t}}\right)$$

$$=\frac{1}{\sum_{i=1}^{n} X_{i,t+1}} \sum_{i=1}^{n} \frac{X_{i,t+1}X_{i,t+1}}{X_{i,t}}$$

$$=\frac{1}{\sum_{i=1}^{n} X_{i,t+1}} \sum_{i=1}^{n} \frac{X_{i,t+1}^{2}}{X_{i,t}}$$

$$=\frac{1}{\sum_{i=1}^{n} X_{i,t+1}} \sum_{i=1}^{n} (r_{i}^{t,t+1} \times X_{i,t+1})$$

2. Arithmetic average of two values of ROXY index

$$\frac{1}{2}\left(\frac{\sum_{i=1}^{n} X_{i,t+1}}{\sum_{i=1}^{n} X_{i,t}} \times \frac{n}{\sum_{i=1}^{n} \frac{X_{i,t+1}}{X_{i,t}}} + \frac{\sum_{i=1}^{n} \frac{X_{i,t+1}^{2}}{X_{i,t}}}{\sum_{i=1}^{n} X_{i,t+1}} \times \frac{n}{\sum_{i=1}^{n} \frac{X_{i,t+1}}{X_{i,t}}}\right)$$

$$=\frac{n}{2\sum_{i=1}^{n} \frac{X_{i,t+1}}{X_{i,t}}}\left(\frac{\sum_{i=1}^{n} X_{i,t+1}}{\sum_{i=1}^{n} X_{i,t}} + \frac{\sum_{i=1}^{n} \frac{X_{i,t+1}^{2}}{X_{i,t}}}{\sum_{i=1}^{n} X_{i,t+1}}\right)$$

$$=\frac{n}{2\sum_{i=1}^{n} r_{i}^{t,t+1}}\left\{\frac{\sum_{i=1}^{n} (r_{i}^{t,t+1} \times X_{i,t})}{\sum_{i=1}^{n} X_{i,t}} + \frac{\sum_{i=1}^{n} (r_{i}^{t,t+1} \times X_{i,t+1})}{\sum_{i=1}^{n} X_{i,t+1}}\right\}$$

(Note) See notes 2, 3 and 4 of Table 5 for notational conventions.

Table N-3 Possible ROXY Index: With Weighting Factor being Average of Population Levels at Different Time Points

$$\sum_{i=1}^{n}\left(\frac{\frac{X_{i,t}+X_{i,t+1}}{2}}{\sum_{j=1}^{n} \frac{X_{j,t}+X_{j,t+1}}{2}} \times \frac{X_{i,t+1}}{X_{i,t}}\right) \times \frac{n}{\sum_{i=1}^{n} \frac{X_{i,t+1}}{X_{i,t}}}$$

$$=\frac{n}{\sum_{i=1}^{n} (X_{j,t}+X_{j,t+1}) \times \sum_{i=1}^{n} \frac{X_{i,t+1}}{X_{i,t}}} \sum_{i=1}^{n} \frac{X_{i,t+1}(X_{i,t}+X_{i,t+1})}{X_{i,t}}$$

$$=\frac{n}{\sum_{i=1}^{n} (X_{i,t}+X_{i,t+1}) \times \sum_{i=1}^{n} r_{i}^{t,t+1}} \sum_{i=1}^{n} \{r_{i}^{t,t+1}(X_{i,t}+X_{i,t+1})\}$$

(Note) See notes 2, 3 and 4 of Table 5 for notational conventions.

Table N-4 Two Possible Indices

WAGRatio (weighting factor: population at T_0)

$$=1.0\times\frac{10}{30}+2.0\times\frac{10}{30}+3.0\times\frac{10}{30}$$
$$=(10+20+30)/30$$
$$=2.0$$

WAGRatio (weighting factor: population at T_1)

$$=1.0\times\frac{10}{60}+2.0\times\frac{20}{60}+3.0\times\frac{30}{60}$$
$$=(10+40+90)/60$$
$$=7/3=2.3333$$

WAGRatio (weighting factor: average of population levels at T_0 and T_1)

$$=1.0\times\frac{10}{45}+2.0\times\frac{15}{45}+3.0\times\frac{20}{45}$$
$$=(10+30+60)/45$$
$$=20/9=2.2222$$

SAGRatio

$$=(1.0+2.0+3.0)/3$$
$$=2.0$$

Possible index (average of two ROXY indicies)

$$=\frac{1}{2}\left(\frac{2.0}{2.0}+\frac{7/3}{2.0}\right)$$
$$=13/12=1.0833$$

Possible index (with weighting factor of average population levels)

$$=\frac{20/9}{2.0}$$
$$=10/9=1.1111$$

weighting factor. The author wishes to thank Takeo Fukuchi and Tony E. Smith for their comments on this point. Another possible approach would be to develop an index defined as follows;

> the ROXY index for the period between time t and time $t+1$ the value of which shall be calculated by employing, as its weighting factor, the average of population levels at time t and time $t+1$ for each spatial unit.

Table N-3 expresses this index in mathematical formula from which we can see that the value of ROXY index (Type I) turns out to be unity if the average of population levels at time t and time $t+1$ is identical for all spatial units. Be that as it may, Table N-4 shows the values of the above-mentioned two kinds of indices for the numerical example shown by Table N-1.

7) No data is available on SMSAs for the period 1940–50.

8) The values of ROXY index (Type II) shown in Table 10 and those to be shown later on in Tables N-5 and N-6, are those values calculated based on the annual growth rates. However, precisely speaking, the following approximation method was applied for their calculation;

WAGRatio (per annum)
=fifth root of "WAGRatio (per pentad)"
or
=tenth root of "WAGRatio (per decade)"*
SAGRatio (per annum)
=fifth root of "SAGRatio (per pentad)"
or
=tenth root of "SAGRatio (per decade)."*

Accordingly, it turns out that;

ROXY index (Type II, per annum)
=[{fifth root of "WAGRatio (per pentad)/SAGRatio (per pentad)"} $-1.0]\times 10^4$
=[{fifth root of "ROXY index (Type I, per pentad)"} $-1.0]\times 10^4$
=[{fifth root of "ROXY index (Type II, per pentad)/10^4 +1.0"}$-1.0]\times 10^4$
or
=[{tenth root of "WAGRatio (per decade)/SAGRatio (per decade)"} $-1.0]\times 10^4$*
=[{tenth root of "ROXY index (Type I, per decade)"} $-1.0]\times 10^4$*
=[{tenth root of "ROXY index (Type II, per decade)/10^4 +1.0"}$-1.0]\times 10^4$*.

This approximation method could be roughly justified on the following grounds. That is, the WAGRatio (per annum) is equal to;

$$\begin{aligned}
&\sum_{i=1}^{n}(r_i^{1/5}X_i)\Big/\sum_{i=1}^{n}X_i \\
&=\sum_{i=1}^{n}\{(1+s_i)^{1/5}X_i\}\Big/\sum_{i=1}^{n}X_i \\
&\simeq\sum_{i=1}^{n}\{(1+s_i/5)X_i\}\Big/\sum_{i=1}^{n}X_i \\
&=\left(\sum_{i=1}^{n}X_i+\sum_{i=1}^{n}s_iX_i/5\right)\Big/\sum_{i=1}^{n}X_i \\
&=1+\sum_{i=1}^{n}s_iX_i\Big/\left(5\sum_{i=1}^{n}X_i\right) \qquad \cdots\cdots(1)
\end{aligned}$$

On the other hand, the fifth root of "WAGRatio (per pentad)" is equal to;

$$\left\{\sum_{i=1}^{n}(r_iX_i)\Big/\sum_{i=1}^{n}X_i\right\}^{1/5}$$

$$=\left[\sum_{i=1}^{n}\{(1+s_i)X_i\}\Big/\sum_{i=1}^{n}X_i\right]^{1/5}$$

$$=\left\{\sum_{i=1}^{n}(X_i+s_iX_i)\Big/\sum_{i=1}^{n}X_i\right\}^{1/5}$$

$$=\left\{\left(\sum_{i=1}^{n}X_i+\sum_{i=1}^{n}s_iX_i\right)\Big/\sum_{i=1}^{n}X_i\right\}^{1/5}$$

$$=\left(1+\sum_{i=1}^{n}s_iX_i\Big/\sum_{i=1}^{n}X_i\right)^{1/5}$$

$$\simeq 1+\sum_{i=1}^{n}s_iX_i\Big/\left(5\sum_{i=1}^{n}X_i\right) \qquad \cdots\cdots(2)$$

As to the SAGRatio (per annum), it is equal to:

$$\sum_{i=1}^{n}r_i^{1/5}/n$$

$$=\sum_{i=1}^{n}(1+s_i)^{1/5}/n$$

$$\simeq\sum_{i=1}^{n}(1+s_i/5)/n$$

$$=\left(n+\sum_{i=1}^{n}s_i/5\right)\Big/n$$

$$=1+\sum_{i=1}^{n}s_i/(5n) \qquad \cdots\cdots(3)$$

On the other hand, the fifth root of "SAGRatio (per pentad)" is equal to;

$$\left(\sum_{i=1}^{n}r_i/n\right)^{1/5}$$

$$=\left\{\sum_{i=1}^{n}(1+s_i)/n\right\}^{1/5}$$

$$=\left(\sum_{i=1}^{n}1/n+\sum_{i=1}^{n}s_i/n\right)^{1/5}$$

$$=\left(1+\sum_{i=1}^{n}s_i/n\right)^{1/5}$$

$$\simeq 1+\sum_{i=1}^{n}s_i/(5n) \qquad \cdots\cdots(4)$$

Therefore, as long as "s_i (for all i)", "$\sum_{i=1}^{n}s_i/n$" and "$\sum_{i=1}^{n}s_iX_i/\sum_{i=1}^{n}X_i$" are all reasonablly small as compared with unity, it can be seen that from (1) and (2)

WAGRatio (per annum) $\simeq$ fifth root of "WAGRatio (per pentad)"

and that from (3) and (4)

SAGRatio (per annum) $\simeq$ fifth root of "SAGRatio (per pentad)."

The notational conventions used in the above argument are as follows;

WAGRatio (per annum), WAGRatio (per pentad), and WAGRatio (per decade):
Weighted average growth ratio calculated based on the annual growth rate, five-year growth rate, and ten-year growth rate respectively,

SAGRatio (per annum), SAGRatio (per pentad), and SAGRatio (per decade):
Simple average growth ratio calculated beased on the annual growth rate, five-year growth rate, and ten-year growth rate respectively,

ROXY index (Type II, per annum), ROXY index (Type II, per pentad), and ROXY index (Type II, per decade):
ROXY index (Type II) calculated based on the annual growth rate, five-year growth rate, and ten-year growth rate respectively,

ROXY index (Type I, per pentad), and ROXY index (Type I, per decade):
ROXY index (Type I) calculated based on the five-year growth rate, and ten-year growth rate respectively,

n : Number of spatial units,
X_i : Population level of spatial unit i at the beginning of a five-year period,
r_i : Five-year growth ratio of population in spatial unit i for the five-year period,
s_i : Five-year growth rate of population in spatial unit i for the five-year period,
* : For the thirty SMSAs as to the period 1960–1970.

9) The "larger" FUCs are those FUCs which are relatively large among the largest thirty FUCs, while the "less-larger" FUCs are those FUCs which are relatively small among the largest thirty FUCs. This way of expression also applies in the remarks of the largest thirty SMSAs in the U.S. as well as in the remarks of the twenty-four central cities in Japan and twenty-nine central cities of the U.S.

10) Table N-5 shows the value of ROXY index (Type II) for the eighty-six FUCs in Japan for the four consecutive five-year periods from 1960 through 1980. It can be pointed out that, throughout the entire twenty-year period 1960–80, the value of ROXY index remains positive. This would imply that in these two decades the FUCs with larger population grew generally faster than those FUCs with smaller population. The value of ROXY index, however, continuously decreases from 121.0 for the period 1960–65 down to approximately zero (viz. 0.5) for the period 1975–80. This would imply that the discrepancy in population growth rates between larger FUCs and smaller FUCs became gradually narrower during the two decades. In other words, the Japanese urban system composed of eighty-six FUCs showed the decelerating concentration of polulation throughout the whole twenty-year period 1960–1980. Taking into account such a trend of changes in the value of ROXY index, it would be quite probable that the ROXY index for the forthcoming five-year period 1980–85 will take a negative value. This would suggest that, for the first time in the postwar period in Japan, the smaller FUCs will begin in the first half of

the 1980s to grow in general faster than the larger FUCs. In this sense, the 1980s could perhaps be viewed as an epochmaking era in the postwar history of the population changes in the urban system of Japan comprising the eighty-six FUCs.

Table N-5 ROXY Index (Type II) for an Urban System of Japanese 86 FUCs

Period / Group of Spatial Units	1960–65	1965–70	1970–75	1975–80
86 FUCs	121.0	84.5	39.5	0.5

(Note) The values of ROXY index shown in this table have been calculated on the basis of the annual growth rates instead of the five-year growth rates.

11) Table N-6 shows the values of ROXY index (Type II) for (i) the group of central cities of the twenty-four FUCs included in the largest thirty FUCs in Japan and (ii) the group of central cities of the twenty-nine SMSAs included in the largest thirty SMSAs in the U.S. It can be pointed out that, for the case of Japan, the value of ROXY index decreases from −56.1 for the period 1960–65 to −119.5 for the period 1965–70. The ROXY index then begins to increase to have the values of −114.6 for the period 1970–75 and −71.3 for the period 1975–80. This would imply that the population growth rates of "larger" central cities of Japan were in general already lower than those of "less-larger" central cities in the first half of the 1960s, and that the speed of population deconcentration in the urban system of the twenty-four central cities was accelerated until the end of that decade. After that, however, the speed of population deconcentration began to be decelerated and this decelerating deconcentration continued throughout the 1970s. For the case of the U.S., the value of ROXY index remains negative in the entire twenty-year period of 1960–80 and continuously increases from —75.5 for the period 1960–70 to —37.0 for the period 1970–75 and then up to —19.9 for the period 1975–80. This would imply, if we can in one way or another assume that the effects of the changes in the boundaries of some of these central cities upon the value of ROXY index would be rather insignificant, that the population deconcentration in the urban system of the twenty-nine central cities in the U.S. has been continuously decelerated since the first half of the 1960s. Summing up the basic contents of Table N-6 together with those of Tables 10 and N-5, we can construct Table N-7 to compare the stages of spatial redistribution of population for three kinds of urban systems in Japan and two kinds of urban systems in the U.S. This table would tell us that the most advanced urban system in a broader sense in light of the theory of spatial cycles (i.e., a theory on urban area's four successive metamorphic stages of urbanization, suburbanization, disurbanization and reurbanization) developed by Klaassen et al. (18), is the one composed of the twenty-nine central cities in the U.S. followed by the urban systems of the thirty SMSAs, the twenty-four central cities in Japan, the thirty FUCs and the eighty-six

FUCs in this order. Meanwhile, for the purpose of overall comparison between the values of ROXY index (Type II) which have been calculated based on the annual growth rates and those calculated based on the five-year growth rates, these two kinds of figures are shown in Table N-8 for the urban systems of eighty-six FUCs, thirty FUCs and twenty-four central cities in Japan as well as for the urban systems of thirty SMSAs and twenty-nine central cities in the U.S. Table N-8 also shows the values of ROXY index (Type II)

Table N-6 ROXY Index (Type II) for the Group of Central Cities (CCs) of Large Metropolitan Areas: Japan vis-à-vis the U.S.

(a) For Japan

Group of Spatial Units \ Period	1960–65	1965–70	1970–75	1975–80
24 CCs	−56.1	−119.5	−114.6	−71.3

(b) For the U.S.

Group of Spatial Units \ Period	1960–70	1970–75	1975–80
29 CCs	−75.5	−37.0	−19.9

(Note) 1. The boundaries of the central cities are as of 1980 and fixed over the time for the case of Japan.
2. The boundaries of the central cities are not fixed but variable over the time for the case of the U.S.
3. The values of ROXY index shown in this table have been calculated on the basis of the annual growth rates.

Table N-7 Pattern of the Spatial Redistribution of Population for Five Urban Systems

Country	Urban System	Spatial Redistribution of Population: Pattern of Dynamic Change for the Second Half of the 1970s	Value of ROXY Index (Type II)	Rank of Urban Advancement
Japan	86 FUCs	Decelerating Concentration (but Close to Stable Share)	0.5	5
	30 FUCs	Accelerating Deconcentration	−19.3	4
	24 CCs	Decelerating Deconcentration	−71.3	3
U.S.	30 SMSAs	Decelerating Deconcentration	−53.5	2
	29 CCs	Decelerating Deconcentration	−19.9	1

(Note) 1. FUC stands for Functional Urban Core.
2. SMSA Stands for Statistical Standard Metropolitan Area.
3. CC stands for central city.

Table N-8 ROXY index (Type II) calculated based on the Annual and Five-year Growth Rates

ROXY index	Country	Group of Spatial Units \ Period	1960–65	1965–70	1970–75	1975–80
ROXY index (Type II, per annum)	Japan	86 FUCs	121.0	84.5	39.5	0.5
		30 FUCs	89.2	42.8	−6.1	−19.3
		24 CCs	−56.1	−119.5	−114.6	−71.3
	U.S.	30 SMSAs	−68.5		−59.0	−53.5
		29 CCs	−75.5		−37.0	−19.9
ROXY index (Type II, per pentad)	Japan	86 FUCs	620.0	429.6	199.1	2.4
		30 FUCs	453.8	215.7	−31.2	−96.1
		24 CCs	−277.5	−583.3	−560.0	−351.3
	U.S.	30 SMSAs	−338.2	(−664.9)	−291.7	−264.6
		29 CCs	−371.9	(−730.0)	−184.0	−99.1

(Note) 1. For notational conventions, see Table 7 and Note 8.
2. Figers in the parentheses indicate the values of ROXY index (Type II, per decade).

calculated based on the ten-year growth rates for the urban systems of thirty SMSAs and twenty-nine central cities in the U.S. as to the ten-year period 1960–70.

12) For this case, the value of ROXY index shall be calculated by means of the formula shown in Table N-9. From the last equational expression in this table, the following exposition and argument can be drawn in conjunction with the relationships between the value of ROXY index (Type II) and the value of the coefficient of the explaining variable in a simple regression line obtained through the ordinary least squares (OLS) estimation method. That is, the ROXY index (Type II) is equal to;

$$\left(\frac{\sum_{i=1}^{n} d_i r_i^{t,t+1}}{\sum_{i=1}^{n} d_i} \times \frac{n}{\sum_{i=1}^{n} r_i^{t,t+1}} - 1.0\right) \times 10^4$$

$$= \frac{n\sum_{i=1}^{n} d_i r_i^{t,t+1} - \sum_{i=1}^{n} d_i \times \sum_{i=1}^{n} r_i^{t,t+1}}{\sum_{i=1}^{n} d_i \times \sum_{i=1}^{n} r_i^{t,t+1}} \times 10^4$$

On the other hand, the OLS estimate of the coefficient b for the regression equation

$$r = a + b \times d$$

where

r : population growth ratio
d : distance from central business district
a, b : regression coefficients,

is equal to;

$$\frac{n\sum_{i=1}^{n} d_i r_i^{t,t+1} - \sum_{i=1}^{n} d_i \times \sum_{i=1}^{n} r_i^{t,t+1}}{n\sum_{i=1}^{n} d_i^2 - \left(\sum_{i=1}^{n} d_i\right)^2}$$

Hence, it follows;

(i) that the value of ROXY index (Type II) turns out to be greater than, equal to, or less than zero if and only if the value of the coefficient b is greater than, equal to, or less than zero respectively,

(ii) that the ROXY index (Type II) and the ROXY index (Type I) as well are "scale-invariant" in the sense that their values are independent of the applied scale-unit because the ROXY index is physically dimensionless, while the coefficient b has the dimension of $[r \times d^{-1}]$, and

(iii) that, when all d_i's are identical or nearly identical to each other, the value of ROXY index (Type II) falls in or around zero (unless $d_i=0$ and $r_i=0$ for all i) while the coefficient b will become quite unstable in the sense that it either will become impossible to calculate, will have extremely high positive value, or will have extremely low negative value.

In the meantime, Table N-10 illustrates a hypothetical example in which the population in the urban center of a specific metropolitan area is steadily declining while its suburban

Table N-9 ROXY Index with Weighting Factor of Distance to Central Business District

1. Weighted average growth ratio (WAGRatio)

$$\sum_{i=1}^{n}\left(\frac{d_i}{\sum_{j=1}^{n} d_j} \times r_i^{t,t+1}\right)$$

2. Simple average growth ratio (SAGRatio)

$$\frac{\sum_{i=1}^{n} r_i^{t,t+1}}{n}$$

3. ROXY index (Type II)

$$(\text{WAGRatio}/\text{SAGRatio} - 1.0) \times 10^4$$

$$= \left(\frac{\sum_{i=1}^{n} d_i r_i^{t,t+1}}{\sum_{i=1}^{n} d_i} \times \frac{n}{\sum_{i=1}^{n} r_i^{t,t+1}} - 1.0\right) \times 10^4$$

(Note) 1. d_i: Distance from subarea i to central business district.
2. n: Number of subareas.
3. $r_i^{t,t+1}$: Growth ratio of population in subarea i for the period between time t and $t+1$.

Table N-10 ROXY Index (Type II) for the Analysis of Suburbanization: Numerical Example

1. Growth ratio

Average Distance to CBD	Subarea \ Period	$T_0 \sim T_1$	$T_1 \sim T_2$
2 km	Inner Ring	0.9	0.8
4 km	Outer Ring	1.2	1.5

2. ROXY index

Period	$T_0 \sim T_1$	$T_1 \sim T_2$
Weighted Average Growth Ratio	1.1000	1.2667
Simple Average Growth Ratio	1.0500	1.1500
ROXY Index (Type II)	476	1014

(Note) 1. CBD: Central business district.
2. Inner ring: Central city.
3. Outer ring: Suburbs.

population is rapidly increasing. As can be easily expected, the value of ROXY index (Type II) for this example remains positive and continues to increase over the time.

13) For example, in case we carry out an analysis on the spatial redistribution of polpulation as to the issues of aging-society, it might be useful to employ as weighting factor "the mean age of the regional total population" or "percentage share of the oldage population (i.e., those who are at the age of sixty-five years and over) against the total regional population."

REFERENCES

(1) Atkinson, A. B. "On the Measurement of Inequality", *Journal of Economic Theory*, Vol. 2, 1970.
(2) Bachi, R. *Statistical Analysis of Geographical Series*, Kaplan School of Hebrew University and Israel Central Bureau of Statistics, Jerusalem, 1957.
(3) Blackorby, C., D. Donaldson and J. A. Weymark. "A New Theory of Concentration Indices and Indicators of Industrial Performance", Discussion Paper No. 79–34, Department of Economics, University of British Columbia, October 1978.
(4) Creamer, D. "Shifts of Manufacturing Industries" in *Industrial Location and National Resources*, U.S. National Resources Planning Board, Washington D. C., 1943.
(5) Davies, S. "Choosing Between Concentration Indices: The Iso-Concentration Curve", *Economica*, Vol. 46, 1979.
(6) Duncan, O. D. "Urbanization and Retail Specialization", *Social Forces*, Vol. 30, March 1952.
(7) ——— and B. Duncan. "Residential Distribution and Occupational Stratification", *American Journal of Sociology*, Vol. 60, March 1955.
(8) ——— and ———. "A Methodological Analysis of Segregation Indexes", *American Sociological Review*, Vol. 20, April 1955.

(9) Finkelstein, M. and R. Friedberg. "The Application of an Entropy Theory of Concentration to the Clayton Act", *Yale Law Journal*, Vol. 76, 1967.
(10) Florence, P. S., W. G. Fritz and R. C. Gilles. "Measures of Industrial Distribution" in *Industrial Location and National Resources*, U.S. National Resources Planning Board, Washington D. C., 1943.
(11) Gibra, R. *Les Inégalités Économiques*, Paris: Sirey, 1931.
(12) Gini, C. "On the Measure of Concentration with Special References to Income and Wealth" in *Abstracts of Papers Presented at the Cowles Commission Research Conference on Economics and Statistics*, Colorado: Colorado College Press, 1936.
(13) Hart, P. "Entropy and Other Measures of Concentration", *Journal of the Royal Statistical Society*, Series A, Vol. 134, 1971.
(14) Herfindahl, O. "Concentration in the U.S. Steel Industry", Unpublished Doctoral Dissertation, Columbia, 1950.
(15) Hirschman, A. "The Paternity on an Index," *American Economic Review*, Vol. 54, 1964.
(16) Hoover, E. M. "Redistrinbution of Population, 1850–1940", *The Journal of Economic History* Vol. 1, November 1941.
(17) Kawashima, T. "Recent Urban Trends in Japan: Analysis of Functional Urban Regions" in T. Kawashima and P. Korcelli (eds.), *Human Settlement Systems: Spatial Patterns and Trends*, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1982.
(18) Klaassen, L. H. et al. *Transport and Reurbanization*, Hants, England: Gower, 1981.
(19) Lorenz, M. O. "Methods of Measuring the Concentration of Wealth", *Publications of the American Statistical Association, New Series*, No. 70, 1905.
(20) Rodgers, A. "Some Aspects of Industrial Diversification in the United States", *Papaers and Proceedings of the Regional Science Association*, Vol. 1, 1955.
(21) ———, "Some Aspects of Industrial Diversification in the United States", *Economic Geography*, Vol. 33, January 1957.
(22) Theil, H. *Economics and Information Theory*, Amsterdam: North-Holland, 1967.
(23) Zelinsky, W. "A Method for Measuring Change in the Distribution of Manufacturing Industry: The United States, 1939–47", *Economic Geography*, Vol. 34, April 1958.