Mathematical Characteristics of ROXY Index (Ⅲ): Functional Relationship between "Theoretically-ideal ROXY Index with CBD Distance Used as Weighing Factor" and "That with Reversed CBD Distance"

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For the quantitative intra-metropolitan analysis in the field of urban and regional economics, there are usually two kinds of approaches in obtaining the value of the ROXY index. One is to calculate the ROXY index by the use of the CBD distance as its weighing factor, while the other is to calculate the ROXY index by the use of the reversed CBD distance as its weighing factor. This paper first shows, for each of the theoretically-ideal continuous-linear and fan-shaped regions, the functional relationship between the values of the two ROXY indices calculated through the above two types of approaches. Then, examining the values of the ROXY index (with the reversed CBD distance used as its weighing factor) calculated for the three systems of the discrete-linear, continuous-linear and fan-shaped regions, the paper discusses (i) the dynamic structural features of the spatial-cycle path of the five railway-line regions in the Tokyo metropolitan area and (ii) the mathematical characteristics of the values of the ROXY index for the three systems of the regions.

Key Words

CBD distance, Continuous-linear region, Discrete-linear region, Fan-shaped region, Railway-line region, Reversed CBD distance, ROXY index, Spatial cycles, Suburbanization, Tokyo metoropolitan area, and Urbanization

1 Introduction

In the investigation of the stages and periods of the spatial-cycle paths for various socio-economic activities, the ROXY index has played a relatively significant role as an analytical instrument in a number of empirical studies since its first application by Kawashima (1978). In parallel have also been carried out theoretical investigations into the mathematical characteristics of the ROXY index. For instance, examining the conventional formulation of the ROXY index which is conceptually constructed for the one-dimensional discrete-linear region, Kawashima and Hiraoka (1993) theoretically clarified the existence of the functional relationship *between* the value of the ROXY-index for which the CBD distance is used as its weighing factor *and* that for which the reversed CBD distance is used as its weighing factor[1].

In their recent study, meanwhile, Hiraoka and Kawashima (1993) developed two types of theoretically-ideal formulations of the ROXY index. One is constructed for the one-dimensional continuous-linear region, while the other is for the two-dimensional fan-shaped region. Along the extension line of this study, for the two types of theoretically-ideal ROXY indices, the present paper examines the functional relationship *between* the value of the ROXY index with the CBD distance used as its weighing factor *and* that with the reversed CBD distance used as its weighing factor.

For this purpose, we employ following notational conventions in our examination;

d_0 : Distance from the CBD to its nearest locality,

d_1 : Distance from the CBD to its farthest locality,

θ_0 : Half of the central angle of the two-dimensional fan-shaped region,

$r(x)$: Annual growth ratio of population at distance x from the CBD in the one-dimensional continuous-liner region,

$r(x,\theta)$: Annual growth ratio of population at distance x from the CBD and angle θ in the two-dimensional fan-shaped region.

2 Functional Relationship between Two ROXY Indices: For Continuous-linear Region

We begin with R_L which represents the value of the normalized ROXY index with the CBD distance used as its weighing factor for the one-dimensional continuous-linear region. The weighted-average growth ratio (WAGR) and simple-average growth ratio (SAGR) for R_L, are respectively calculated as follows;

WAGR (Weighted-average Growth Ratio)

$$= \frac{\int_{d_0}^{d_1} x r(x) dx}{\int_{d_0}^{d_1} x dx}$$

$$= \frac{2}{d_1^2 - d_0^2} \int_{d_0}^{d_1} x r(x) dx \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(1)}$$

SAGR (Simple-average Growth Ratio)

$$= \frac{\int_{d_0}^{d_1} r(x) dx}{\int_{d_0}^{d_1} dx}$$

$$= \frac{1}{d_1 - d_0} \int_{d_0}^{d_1} r(x) dx \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(2)}$$

Denoting $(d_1 - d_0)/(d_1 + d_0)$ by α $(\alpha > 0)$, from equations (1) and (2) we have the following expression[2];

$$\alpha R_L \times 10^{-4} = \frac{WAGR}{SAGR} - 1.0$$

$$= \frac{2}{d_1^2 - d_0^2} \cdot \int_{d_0}^{d_1} x \cdot r(x) dx \cdot \frac{d_1 - d_0}{\int_{d_0}^{d_1} r(x) dx} - 1.0$$

$$= \frac{2}{d_1 + d_0} \cdot \frac{\int_{d_0}^{d_1} x r(x) dx}{\int_{d_0}^{d_1} r(x) dx} - 1.0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(3)}$$

Meanwhile, R_{LR} which represents the value of the normalized ROXY index with the reversed CBD distance used as its weighing factor for the one-dimensional continuous-linear region, can be calculated as follows in which we define the reversed CBD distance as "s-x" where the constant s is supposed to be independent of the CBD distance x;

$$WAGR = \frac{\int_{d_0}^{d_1} (s - x) r(x) dx}{\int_{d_0}^{d_1} (s - x) dx}$$

$$= \frac{s \int_{d_0}^{d_1} r(x) dx - \int_{d_0}^{d_1} x r(x) dx}{s \int_{d_0}^{d_1} dx - \int_{d_0}^{d_1} x dx}$$

$$= \frac{s \int_{d_0}^{d_1} r(x)dx - \int_{d_0}^{d_1} xr(x)dx}{s(d_1 - d_0) - \frac{1}{2}(d_1^2 - d_0^2)}$$

$$= \frac{s \int_{d_0}^{d_1} r(x)dx - \int_{d_0}^{d_1} xr(x)dx}{\frac{1}{2}(d_1 - d_0)\{2s - (d_1 + d_0)\}} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (4)$$

$$SAGR = \frac{\int_{d_0}^{d_1} r(x)dx}{\int_{d_0}^{d_1} dx}$$

$$= \frac{1}{d_1 - d_0} \int_{d_0}^{d_1} r(x)dx \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (5)$$

From equations (4) and (5), we have the following expression;

$$aR_{LR} \times 10^{-4} = \frac{WAGR}{SAGR} - 1.0$$

$$= \frac{s \int_{d_0}^{d_1} r(x)dx - \int_{d_0}^{d_1} xr(x)dx}{\frac{1}{2}(d_1 - d_0)\{2s - (d_1 + d_0)\}} \cdot \frac{d_1 - d_0}{\int_{d_0}^{d_1} r(x)dx} - 1.0$$

$$= \frac{2s}{2s - (d_1 + d_0)} - \frac{2}{2s - (d_1 + d_0)} \cdot \frac{\int_{d_0}^{d_1} xr(x)dx}{\int_{d_0}^{d_1} r(x)dx} - 1.0 \qquad \cdots\cdots \quad (6)$$

Eliminating two integrals appearing in each of equations (3) and (6), we get

$$aR_{LR} \times 10^{-4} = \frac{2s}{2s - (d_1 + d_0)} - \frac{d_1 + d_0}{2s - (d_1 + d_0)} \times (aR_L \times 10^{-4} + 1.0) - 1.0 \qquad \cdots\cdots \quad (7)$$

If we specify that s is equal to $d_1 + d_0$ so that we have the reversed CBD distance defined as "$d_1 + d_0 - x$" which is definitionally identical to the reversed CBD distance originally employed by Kawashima (1978), then from equation (7) we have

$$aR_{LR} \times 10^{-4} = \frac{2s}{2s - s} - \frac{s}{2s - s} \cdot (aR_L \times 10^{-4} + 1.0) - 1.0$$

$$= 2.0 - (aR_L \times 10^{-4} + 1.0) - 1.0$$

$$= -aR_L \times 10^{-4} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (8)$$

Consequently, we obtain from equation (8) the functional relationship between R_L and R_{LR} for the one-dimensional continuous-linear region that can be expressed as;

$$R_{LR} = -R_L \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (9)$$

Note here again that in equation (9), the reversed CBD distance is defined as "$d_1 + d_0 - x$."

3 Funcional Relationship between Two ROXY Indices: For Fan-shaped Region

Let us denote by R_F the value of the normalized ROXY index with the CBD distance used as its weighing factor for the two-dimensional fan-shaped region. Then, WAGR and SAGR are respectively calculated as follows;

$$
\begin{aligned}
WAGR &= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x^3 r(x,\theta)d\theta dx}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x^3 d\theta dx} \\[2mm]
&= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x^3 r(x,\theta)d\theta dx}{2\theta_0 \cdot \frac{1}{4}(d_1^4 - d_0^4)} \\[2mm]
&= \frac{2\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x^3 r(x,\theta)d\theta dx}{\theta_0(d_1^2 - d_0^2)(d_1^2 + d_0^2)} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (10)
\end{aligned}
$$

$$
\begin{aligned}
SAGR &= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x d\theta dx}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x d\theta dx} \\[2mm]
&= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x d\theta dx}{2\theta_0 \cdot \frac{1}{2}(d_1^2 - d_0^2)} \\[2mm]
&= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x d\theta dx}{\theta_0(d_1^2 - d_0^2)} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (11)
\end{aligned}
$$

Denoting $(d_1^2 - d_0^2)/(d_1^2 + d_0^2)$ by β $(\beta > 0)$, from equations (10) and (11) we have the following expression[3];

$$\beta R_F \times 10^{-4} \quad = \frac{WAGR}{SAGR} - 1.0$$

$$= \frac{2\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{\theta_0(d_1^2 - d_0^2)(d_1^2 + d_0^2)} \cdot \frac{\theta_0(d_1^2 - d_0^2)}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx} - 1.0$$

$$= \frac{2}{d_1^2 + d_0^2} \cdot \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx} - 1.0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

Meanwhile, R_{FR} which represents the value of the normalized ROXY index with the reversed CBD distance used as its weighing factor for the two-dimensional fan-shaped region, can be calculated as follows in which we define the reversed CBD distance as "$s-x^2$" where the constant s is supposed to be independent of the CBD distance x;

$$WAGR \quad = \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0}(s - x^2)r(x,\theta)xd\theta dx}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0}(s - x^2)xd\theta dx}$$

$$= \frac{s\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx - \int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{s\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} xd\theta dx - \int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} x^3 d\theta dx}$$

$$= \frac{s\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx - \int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{2\theta_0\left(s\int_{d_0}^{d_1} xdx - \int_{d_0}^{d_1} x^3 dx\right)}$$

$$= \frac{s\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx - \int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{2\theta_0\left\{\frac{s}{2}(d_1^2 - d_0^2) - \frac{1}{4}(d_1^4 - d_0^4)\right\}}$$

$$= \frac{s\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx - \int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{\frac{1}{2}\theta_0(d_1^2 - d_0^2)\{2s - (d_1^2 + d_0^2)\}} \qquad \cdots\cdots\cdots\cdots\cdots (13)$$

$$SAGR = \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx}{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} xd\theta dx}$$

$$= \frac{\int_{d_0}^{d_1}\int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx}{\theta_0(d_1^2 - d_0^2)} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (14)$$

From equations (13) and (14) we have the following expression;

$$\beta R_{FR} \times 10^{-4}$$

$$= \frac{WAGR}{SAGR} - 1.0$$

$$= \frac{s \int_{d_0}^{d_1} \int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx - \int_{d_0}^{d_1} \int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{\frac{1}{2}\theta_0(d_1^2 - d_0^2)\{2s - (d_1^2 + d_0^2)\}} \cdot \frac{\theta_0(d_1^2 - d_0^2)}{\int_{d_0}^{d_1} \int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx} - 1.0$$

$$= \frac{2s}{2s - (d_1^2 + d_0^2)} - \frac{2}{2s - (d_1^2 + d_0^2)} \cdot \frac{\int_{d_0}^{d_1} \int_{-\theta_0}^{\theta_0} r(x,\theta)x^3 d\theta dx}{\int_{d_0}^{d_1} \int_{-\theta_0}^{\theta_0} r(x,\theta)xd\theta dx} - 1.0 \qquad \cdots \cdots \cdots \text{(15)}$$

Eliminating two integrals appearing in each of equations (12) and (15), we get

$$\beta R_{FR} \times 10^{-4} = \frac{2s}{2s - (d_1^2 + d_0^2)} - \frac{d_1^2 + d_0^2}{2s - (d_1^2 + d_0^2)} \times \left(\beta R_F \times 10^{-4} + 1.0\right) - 1.0$$

$$\cdots \cdots \cdots \text{(16)}$$

If we specify that s is equal to $d_1^2 + d_i^2$. then from equation (16) we have

$$\beta R_{FR} \times 10^{-4} = \frac{2s}{2s - s} - \frac{s}{2s - s}\left(\beta R_F \times 10^{-4} + 1\right) - 1$$

$$= 2 - \left(\beta R_F \times 10^{-4} + 1.0\right) - 1.0$$

$$= -\beta R_F \times 10^{-4} \qquad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \text{(17)}$$

Consequently, we obtain from equation (17) the functional relationship between R_F and R_{FR} for the two-dimensional fan-shaped region that can be expressed as;

$$R_{FR} = -R_F \qquad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \text{(18)}$$

Note here again that in equation (18), the reversed CBD distance is defined as "$d_1^2 + d_0^2 - x^2$."

4 Empirical Results and Invenstigations for Five Railway-line Regions in Tokyo Metropolitan Area: Discrete-linear, Continuous-linear and Fan-shaped Regions

With the knowledge we have acquired in previous sections on the functional relationships expressed by equations (N-1), (9) and (18), we empirically examine in this section the values of the ROXY index for (i) the discrete-linear region, (ii) the continuous-linear region and (iii) the fan-shaped region, in order to analyze the dynamic structural features of the spatial-cycle path of the five major commuting railway-line regions

in the Tokyo metropolitan area; the Chuo-line, Takasaki-line, Joban-line, Tokaido-line and Sobu-line regions. In our examination, we set two cases with respect to the way of spatial demarcation as to localities constituting each of the railway-line regions; aggregated case and disaggregated case. The cities of Tokyo, Kawasaki and Yokohama are individually considered as a spatial unit in the aggregated case, while in the disaggregated case each of wards (ku) into which each of the above cities are subdivided is individually considered as a separate independent spatial unit[4].

With respect to the aggregated case in the system of the discrete-linear region for the five railway-line regions, Table 1(a) shows the values of the ROXY-indx for which the CBD distance is used as its weighing factor, while Table 1(b) shows those for which the reversed CBD distance is used as its weighing factor. For the Chuo-line region, the ratio of any value in Table 1(b) to its corresponding value appearing in Table 1(a) is always equal to -0.81[5]. For the Takasaki-line, Joban-line, Tokaido-line and Sobu-line regions, the ratios of the two values corresponding to each other in Tables 1(b) and 1(a) are respectively equal to -1.03, -1.28, -1.22 and -1.12. It is to be noted that the absolute value of the ratio for the Chuo-line region is less than 1.0. For each of other railway-line regions, the absolute value of the ratio is greater than 1.0. What has been pointed out in this paragraph would imply the followings;

(1) For the Chuo-line region, the averge of the CBD distance is less than $(d_0 + d_1)/2$[6], which implies that the pattern of the spatial distribution of its localities is skewed towards the inner side of the midpoint between d_0 and d_1.

Table 1 ROXY Index for Discrete-linear Region: Aggregated Case

(a) ROXY index by CBD distance for five railway-line regions in Tokyo metropolitan area

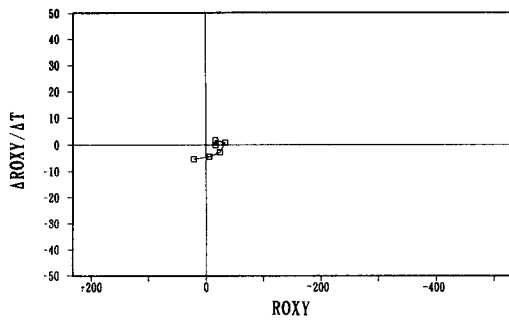| Spatial unit	Number of localities	Period					
		1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	11	-25.45	7.83	29.69	41.84	19.89	20.87
Takasaki-line region	12	-29.49	35.23	41.46	46.77	28.65	18.07
Joban-line region	9	-84.21	-14.43	26.87	56.75	39.06	32.95
Tokaido-line region	6	67.69	77.18	64.62	47.04	18.44	14.84
Sobu-line region	9	-83.82	-17.95	49.03	85.06	62.77	57.17

(b) ROXY index by reversed CBD distance for five railway-line regions in Tokyo metoroporitan area

Spatial unit	Number of localities	Period					
		1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	11	20.73	-6.38	-24.19	-34.08	-16.20	-17.00
Takasaki-line region	12	30.44	-36.37	-42.79	-48.27	-29.57	-18.65
Joban-line region	9	108.13	18.53	-34.50	-72.86	-50.15	-42.31
Tokaido-line region	6	-82.59	-94.16	-78.84	-57.39	-22.49	-18.11
Sobu-line region	9	93.61	20.04	-54.75	-95.00	-70.10	-63.84

(2) For each of the Takasaki-line, Joban-line, Tokaido-line and Sobu-line regions, the average of the CBD distance is greater than $(d_0 + d_1)/2$, which implies that the pattern of the spatial distribution of its localities is skewed towards the outer side of the midpoint between d_0 and d_1.

(3) As compared with the scale of the diagrammatic image of the spatial-cycle path illustrated by means of the circular-cyclic form for the value of the ROXY index with the CBD distance used as its weighing factor, the diagrammatic image for the value of the ROXY index with the reversed CBD distance used as its weighing factor is reduced to the size of 0.81 for the Chuo-line region, and is magnified to the size of 1.03 for the Takasaki-line region, 1.28 for the Joban-line region, 1.22 for the Tokaido-line region and 1.12 for the Sobu-line region.

With respect to the disaggregated case in the system of the discrete-linear region, Table 2(a) shows the values of the ROXY-index with the CBD distance used as its weighing factor, while Table 2(b) shows those with the reversed CBD distance used as its weighing factor. It can be seen that the ratio of any ROXY-index value appearing in Table 2(b) to its corresponding value in Table 2(a) is equal to -0.60 for the Chuo-line region, -0.89 for the Takasaki-line region, -0.94 for the Joban-line region, -0.84 for the Tokaido-line region and -0.70 for the Sobu-line region. For all railway-line regions, the absolute value of each of these ratios is less than 1.0. What has been pointed out in this paragraph would imply the followings;

Table 2   ROXY Index for Discrete-linear Region: Disaggregated Case

(a) ROXY index by CBD distance for five railway-line regions in Tokyo metropolitan area

Spatial unit	Number of localities	Period					
		1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	16	150.12	139.02	117.34	97.51	55.90	107.03
Takasaki-line region	14	36.20	122.95	106.86	90.51	56.55	50.25
Joban-line region	12	-11.27	100.44	145.65	148.81	89.28	70.71
Tokaido-line region	14	171.39	175.06	147.06	88.43	47.36	81.38
Sobu-line region	14	50.51	160.06	210.90	204.64	126.75	140.41

(b) ROXY index by reversed CBD distance for five railway-line regions in Tokyo metoroporitan area

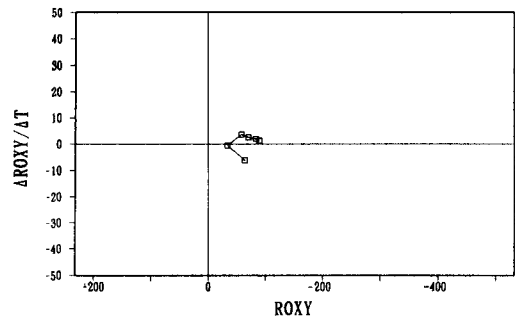| Spatial unit	Number of localities	Period					
		1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	16	-90.03	-83.37	-70.37	-58.48	-33.52	-64.19
Takasaki-line region	14	-32.36	-109.90	-95.52	-80.90	-50.55	-44.92
Joban-line region	12	10.54	-93.93	-136.20	-139.16	-83.48	-66.12
Tokaido-line region	14	-143.79	-146.89	-123.37	-74.19	-39.74	-68.27
Sobu-line region	14	-35.46	-112.37	-148.06	-143.67	-88.98	-98.58

(1)For each railway-line region, the average of the CBD distance is less than $(d_0+d_1)/2$, which implies that the pattern of the spatial distribution of its localities is skewed towards the inner side of the midpoint between d_0 and d_1.

(2)As compared with the scale of the diagrammatic image of the spatial-cycle path illustrated by means of the circular-cyclic form for the value of the ROXY index with the CBD distance used as its weighing factor, the diagrammatic image for the value of the ROXY index with the reversed CBD distance used as its wrighing factor is reduced to the size of 0.60 for the Chuo-line region, 0.89 for the Takasaki-line region, 0.94 for the Joban-line region, 0.84 for the Tokaido-line region and 0.70 for the Sobu-line region.

Meanwhile, as shown in Kawashima and Hiraoka (1993), the values of the average of the CBD distance in the aggergated and disaggregated cases are respectively 28.2km and 21.2km for the Chou-line region, 33.2km and 29.4km for the Takasaki-line region, 31.2km and 25.2km for the Joban-line region, 31.6km and 23.8km for the Tokaido-line region and 30.4km and 21.6km for the Sobu-line region. Since the averege of the CBD distance in the aggregated case is greater than that in the disaggregated case for all the railway-line regions, the stage of the spatial-cycle path for any given time-period would generally be more advanced in the disaggregated case than in the aggregated case[7] for each of the five railway-line regions. This general tendency is reasonably well exhibited by panels (a-1) and (a-2) in each of Figures 1 through 5. It can also be pointed out that the gravity center of the spatial-cycle path digrammatically illustrated by means of the circular-cyclic from would be situated toward the more right-hand direction in the panel (a-2) than in the panel (a-1) for each of Figures 1 through 5[8].

(a-1) Discrete-linear region for aggregated case

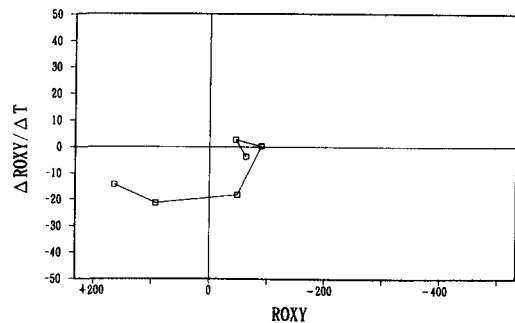(a-2) Discrete-linear region for disaggregated case

(b-1) Continuous-linear region for aggregated case
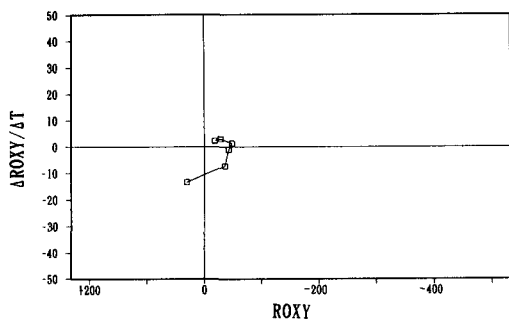
(b-2) Continuous-linear region for disaggregated case

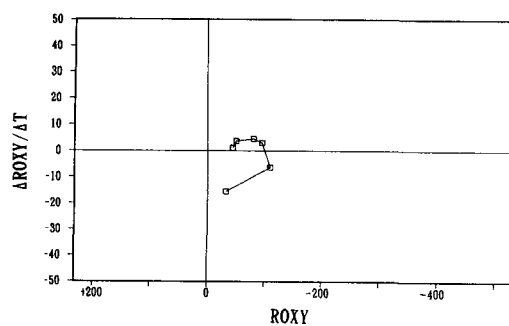(c-1) Fan-shaped region for aggregated case
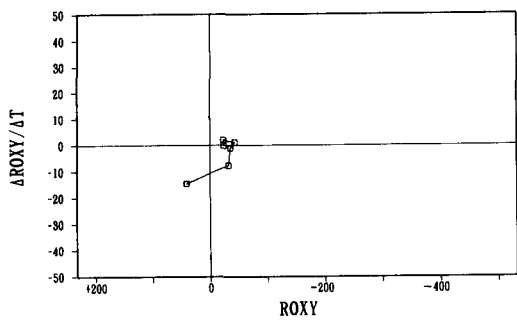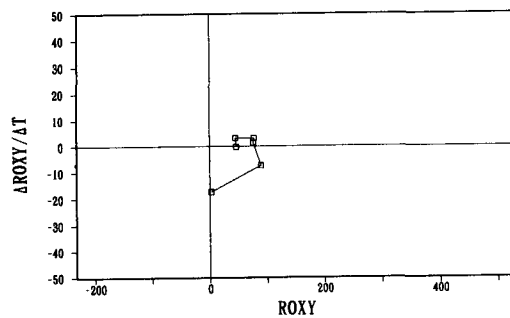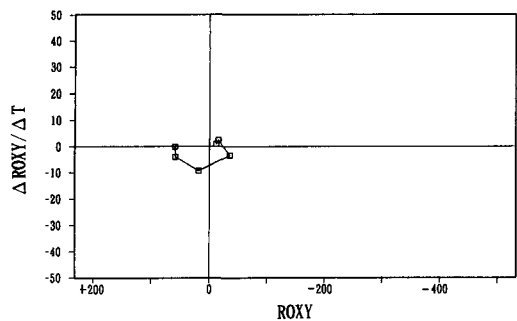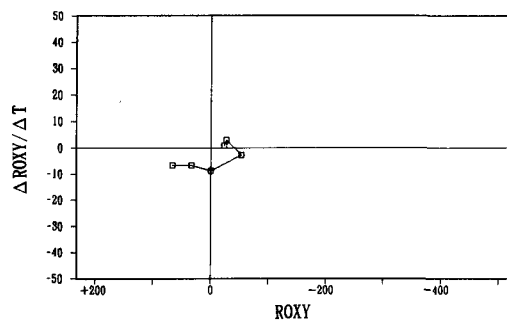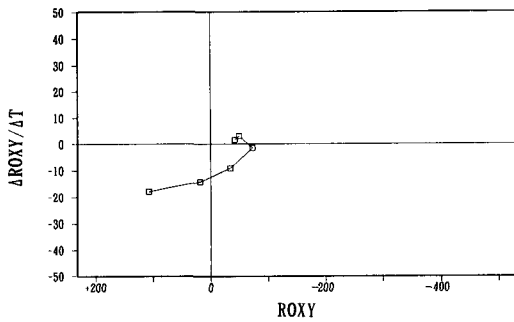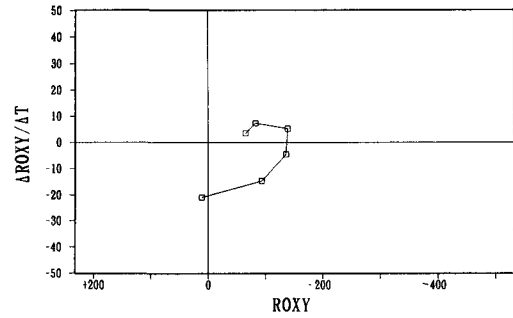
(c-2) Fan-shaped region for disaggregated case

Figure 1   Comparison of ROXY Indices by Reversed CBD Distance: For Chuo–line Region

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(a-1) Discrete-linear region for aggregated case

(a-2) Discrete-linear region for disaggregated case

(b-1) Continuous-linear region for aggregated case

(b-2) Continuous-linear region for disaggregated case

(c-1) Fan-shaped region for aggregated case

(c-2) Fan-shaped region for disaggregated case

**Figure 2   Comparison of ROXY Indices by Reversed
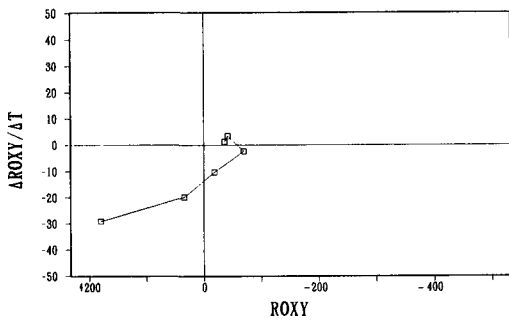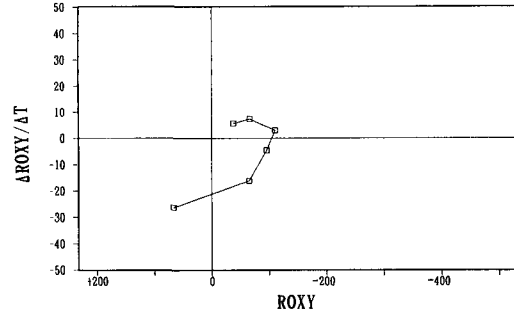CBD Distance: For Takasaki-line Region**

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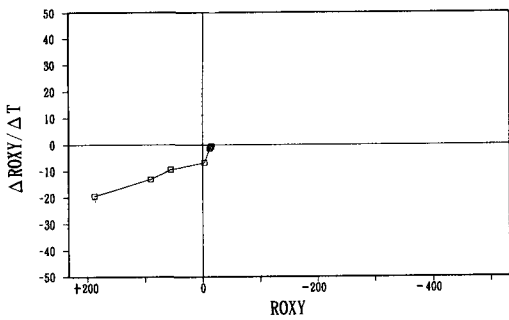(a-1) Discrete-linear region for aggregated case

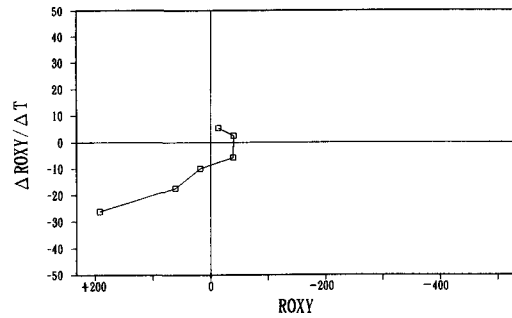(a-2) Discrete-linear region for disaggregated case

(b-1) Continuous-linear region for aggregated case

(b-2) Continuous-linear region for disaggregated case

(c-1) Fan-shaped region for aggregated case

(c-2) Fan-shaped region for disaggregated case

Figure 3　Comparison of ROXY Indices by Reversed CBD Distance: For Joban-line Region

(a-1) Discrete-linear region for aggregated case

(a-2) Discrete-linear region for disaggregated case

(b-1) Continuous-linear region for aggregated case

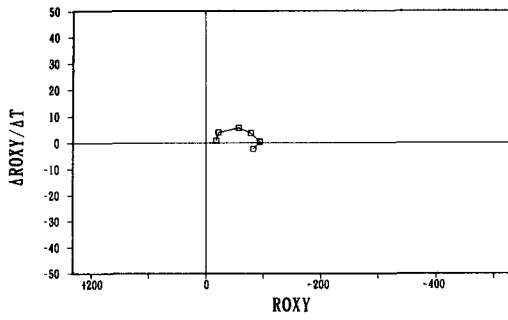(b-2) Continuous-linear region for disaggregated case
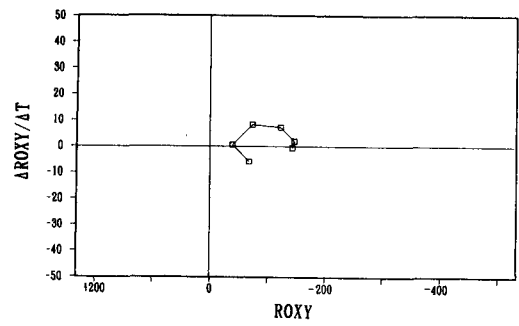
(c-1) Fan-shaped region for aggregated case

(c-2) Fan-shaped region for disaggregated case

**Figure 4　Comparison of ROXY Indices by Reversed
CBD Distance: For Tokaido-line Region**

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(a-1) Discrete-linear region for aggregated case   (a-2) Discrete-linear region for disaggregated case

(b-1) Continuous-linear region for aggregated case   (b-2) Continuous-linear region for disaggregated case

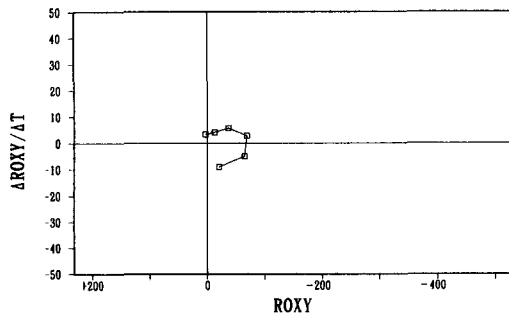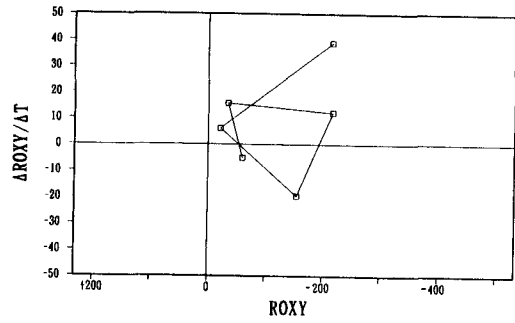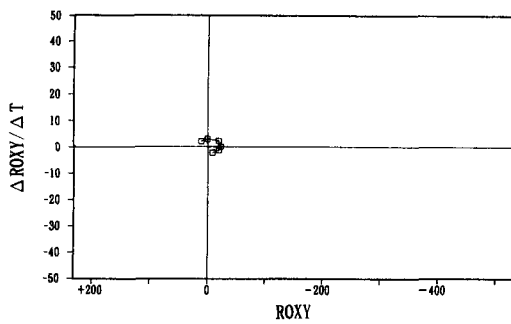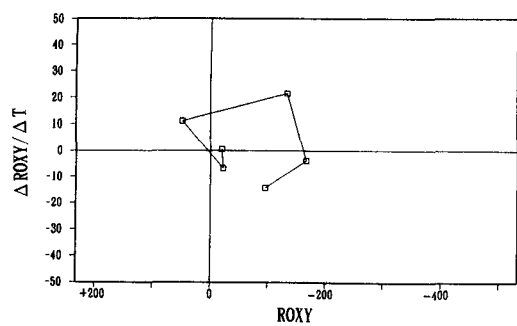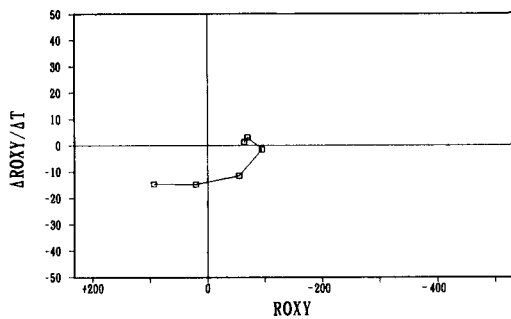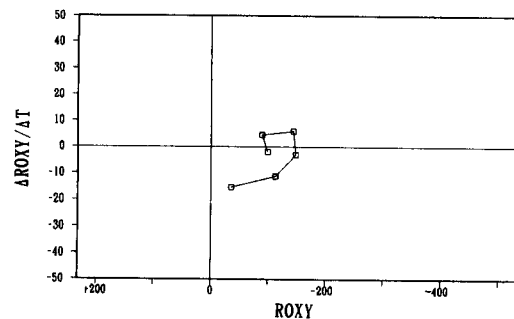(c-1) Fan-shaped region for aggregated case   (c-2) Fan-shaped region for disaggregated case

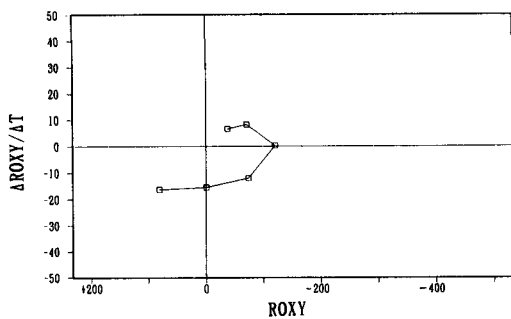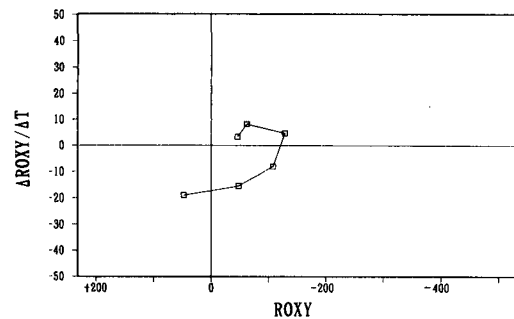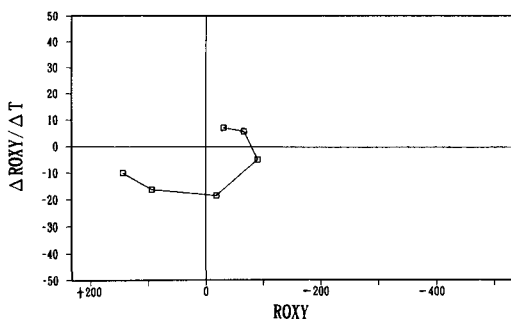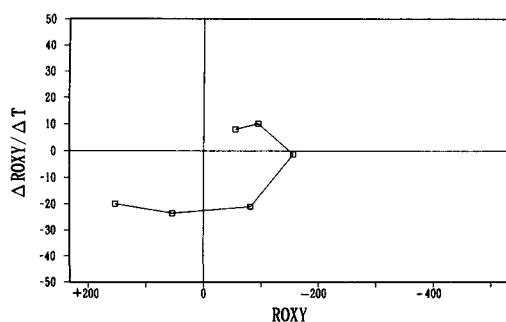**Figure 5   Comparison of ROXY Indices by Reversed CBD Distance: For Sobu-line Region**

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With respect to the aggregated and disaggregated cases in the system of the continuous-linear region for the five raiway-line regions, Tables 3 and 4 respectively show the values of the ROXY index for which the reversed CBD distance is used as its weighing factor[9]. Based on these two tables, panels (b-1) and (b-2) in each of Figures 1 through 5 are obtained. They respectively show the spatial-cycle path diagrammatically illustrated by means of the circular-cyclic form for the aggregated and disaggregated cases. Comparing panels (b-1) with (b-2) for the system of the continuous-linear region, it can be found (i) that the stage of the spatial-cycle path tends to be *a slightly* more advanced in the disaggregated case than in the aggregated case for the Chouo-line, Tokaido-line[10], Sobu-line and Takasaki-line regions, and (ii) that the stage seems to be a slightly less advanced in the disaggregated case than that in the aggregated case for the Joban-line region. The aforementioned would perhaps imply that the values of the ROXY index calculated for the system of the continuous-linear region would be less dependent upon the spatial aggregation and disaggregation of localities as compared with the value of the ROXY-index calculated for the system of the discrete-linear region[11].

Table 3   ROXY Index by Reversed CBD Distance: For Continuous-linear Region in Aggregated Case

Spatial unit	Period					
	1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	94.37	34.78	-55.51	-75.06	-36.18	-40.20
Takasaki-line region	40.80	-32.75	-35.40	-43.63	-23.73	-24.25
Joban-line region	179.56	33.60	-18.98	-69.65	-42.41	-36.37
Tokaido-line region	-20.60	-65.62	-69.39	-37.09	-13.36	3.06
Sobu-line region	80.89	-1.04	-74.26	-120.97	-71.42	-38.55

Table 4   ROXY Index by Reversed CBD Distance: For Continuous-linear Region in Disaggregated Case

Spatial unit	Period					
	1960-65	1965-70	1970-75	1975-80	1980-85	1985-90
Chuo-line region	-1.97	-26.88	-91.99	-95.82	-52.26	-87.12
Takasaki-line region	-3.15	-90.10	-76.35	-77.51	-45.27	-46.54
Joban-line region	66.41	-65.02	-95.93	-110.92	-66.59	-38.10
Tokaido-line region	-216.00	-21.72	-154.90	-216.88	-34.21	-60.18
Sobu-line region	47.40	-47.97	-108.11	-128.15	-61.90	-45.96

With respect to the aggregated and disaggregated cases in the system of the fan-shaped region for the five railway-line regions, Tables 5 and 6 respectively show the values of the ROXY index for which the reversed CBD distance is used as its weighing factor[12]. Based on these two tables, panels (c-1) and (c-2) in each of Figures 1 through 5 are obtained. They respectively show the spatial-cycle path diagrammatically illustrated by means of the circular-cyclic form for the aggregated and disaggregated cases. Comparing panels (c-1) with (c-2), it can be found that, for the system of the fan-shaped region, the *stages* of the spatial-cycle path tend to be more identical between the aggregated and disaggregated cases than the systems of the discrete-linear and continuous-linear regions. This seems to imply that the value of the ROXY index calculated for the system of the fan-shaped region would be more independent of the spatial aggregation and disaggregation of localities than the value of the ROXY-index calculated for the system of the discrete-linear region and that of the continuous—linear region, as long as the spatial aggregation and disaggregation would be carried out for the area *relatively* in the neighgbourhood of the "pivot of the fan" of the fan-shaped region[13].

Table 5   ROXY Index by Reversed CBD Distance: For Fan-shaped Region in Aggregated Case

Spatial unit	Period					
	1960–65	1965–70	1970–75	1975–80	1980–85	1985–90
Chuo-line region	172.15	110.16	−3.13	−41.24	−21.75	−22.19
Takasaki-line region	59.53	58.97	18.42	−35.75	−15.83	−11.83
Joban-line region	186.96	90.06	55.32	−3.60	−12.94	−15.65
Tokaido-line region	−7.85	−19.24	−22.36	−19.65	0.04	10.43
Sobu-line region	143.33	93.17	−19.12	−90.71	−66.81	−30.94

Table 6   ROXY Index by Reversed CBD Distance: For Fan-shaped Region in Disaggregated Case

Spatial unit	Period					
	1960–65	1965–70	1970–75	1975–80	1980–85	1985–90
Chuo-line region	164.81	93.17	−48.90	−89.81	−45.61	−63.61
Takasaki-line region	66.24	33.44	−0.38	−53.59	−27.57	−23.48
Joban-line region	192.00	61.02	17.51	−39.67	−40.51	−13.97
Tokaido-line region	−95.57	−166.61	−133.46	48.02	−22.29	−19.55
Sobu-line region	153.01	53.63	−82.37	−156.22	−95.24	−55.14

Before concluding this section, let us briefly discuss an issue on the boundary between the core and ring areas. This theme of the core-ring boundary is touched on in Hiraoka and Kawashima (1993). Roughly speaking, the CBD distance x of the core-ring boundary is defined as a logical distance satisfying the following condition: If the average growth ratio for the area between d_0 and x is greater than, equal to, or less than that for the area between x and d_1, then the value of the ROXY index would turn out to reflect the stage of centralization, neutrality, or decentralization respectively in the spatial-cycle path. The CBD distance of the core-ring boundary is $\bar{d}$ for the system of the discrete-linear region, $(d_1+d_0)/2$ for the system of the continuous-linear region, and $\{(d_1^2+d_0^2)/2\}^{\frac{1}{2}}$ for the system of the fan-shaped region.

For the aggregated case in our study, we have

$$(d_1+d_0)/2 < \bar{d} < \{(d_1^2+d_0^2)/2\}^{\frac{1}{2}}$$

for all the railway-line regions except the Chuo-line region for which $\bar{d} < (d_1+d_0)/2 < \{(d_1^2+d_0^2)/2\}^{\frac{1}{2}}$. Accordingly, if we employ the CBD distance of the core-ring boundary as a criterion for judging the "relative advancement" along the spatial-cycle path, in the aggregated case the stage of the spatial-cycle path would theoretically tend to be most advanced in the system of the continuous-linear region followed by the system of the discrete-linear region and then by the system of the fan-shaped region. In our empirical studies, however, this theoretical general tendency can be clearly noticed only in Figure 2 for the Takasaki-line region and Figure 5 for the Sobu-line region.

For the disaggregated case, we have

$$\bar{d} < (d_1+d_0)/2 < \{(d_1^2+d_0^2)/2\}^{\frac{1}{2}}$$

for all the railway-line regions. In the disaggregated case, accordingly, the stage of the spatial-cycle path would tend to be most advanced in the system of the discrete-linear region followed by the system of the continuous-linear region and then by the system of the fan-shaped region. This tendency can be clearly noticed if we compare panels (a-2), (b-2) and (c-2) of Figures 1 through 5 except Figure 4 for the Tokaido-line region.

5 Conclusion

Equations (9) and (18) describe that, if we appropriately choose the formulation of the reversed CBD distance, then we can get a simple relationship between the theoretically-ideal ROXY index with the CBD distance used as its weighing factor and that with the reversed CBD distance used as its weighing factor. For both systems of the continuous-linear and fan-shaped regions, the relationship between the values of the ROXY index for these two types of systems can be summarized as follows;

(1) Their absolute values are equal to each other.
(2) Their signs are opposite to each other.

It should be noted that each of the two functional relationships respectively expressed by equations (9) and (18) has no dependency on the pattern of the spatial distribution of lacalities. They depend on neither d_0, d_1, $\overline{d}$, nor $\overline{s}$. This fact is in a striking contrast to the fact that the functional relationship, as expressed by equation (N-1), for the conventional formulation of the ROXY index conceptually set for the one-dimensional discrete-linear region. It should also be noted (i) that, as discussed in notes 11 and 13, the theoretical difference in the spatial-cycle stages between the aggregated and disaggregated cases would be the smallest for the system of the fan-shaped region, the largest for the system of the discrete-linear region, and intermediate between them for the system of the continuous-linear region, (ii) that the system of the continuous-linear region is conceptually more realistic than that of the fan-shaped region when we are interested in the investigation of the centralization and decentralization processes in the railway-line regions, (iii) the calculation of the value of the ROXY index is more mathematically tractable for the system of the discrete-linear region than the continuous-linear and fan-shaped regions, and (iv) that the fact that the *overshooting effects* are dangerously embedded in the ROXY index calculated for the continuous-linear and fan-shaped regions. In the light of the above considerations, it seems to be still premature to conclude which would be in general the best among the three ROXY indices; ROXY indices for (i) discrete-linear region, (ii) continuous-linear region and (iii) fan-shaped region.

Notes

1) The functional relationship between these two ROXY-index values is as follows;

$$R_s = -\frac{\overline{d}}{\overline{s}} \times R_d \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (N-1)$$

where

$\overline{d}$: Average of CBD distance of all localities,

$\overline{s}$: Average of reversed CBD distance of all localities,

R_d : Value of ROXY index which we calculate by use of the CBD distance as its weighing factor,

R_s : Value of ROXY index which we calculate by use of the reversed CBD distance as its weighing factor.

In the above argument, the revered CBD distance is defined as $d_0 + d_1 - d_i$ where d_i, d_0 and d_1 respectively indicate the CBD distance of locality i, the minimum value of d_i, and the maximum value of d_i. The theoretical concept of the reversed CBD distance was initially developed and empirically applied by Kawashima (1987).

2) This expression is derived from equation (4.24) in our previous paper, Hiraoka and Kawashima (1993). In that paper, incidentally, equation (5.7) which can also be derived from equation (4.24) carries errors that should be corrected to what appears in Table A-1(1) in Appendix.

3) This expression is derived from equation (4.36) in Hiraoka and Kawashima (1993). In

that paper, incidentally, equation (5.9) which can also be derived from equation (4.36) carries errors that should be corrected to what appears in Table A-1(2) in Appendix. In pursuance with the new expressions in Table A-1, we have to amend with apologies Tables 22 and 23 as well as Figures 28 and 29 in op. cit. to what appears in Tables A-2(1) and A-2(2) as well as Figures A-1(1) and A-1(2) in Appendix respectively.

4) See Kawashima and Hiraoka (1993) for the more detailed explanations of the aggregated and disaggregated cases for the five vailway-line regions in the Tokyo metropolitan area.

5) As indicated by equation (N-1), this ratio is equal to the ratio of "the average of the CBD distance" to "the average of the reversed CBD distance" multiplied by -1. Note here that in the present case the reversed CBD distance is defined as "$d_1 + d_0 - d_i$."

6) From equation (N-1), if $|R_s| < |R_d|$, then $\bar{d} < \bar{s}$. Hence it follows that

$$\sum_{i=1}^{n} d_i/n \quad < \quad \sum_{i=1}^{n}(d_0 + d_1 - d_i)/n$$

$$\therefore \quad 2\sum_{i=1}^{n} d_i/n \quad < \quad d_0 + d_1$$

$$\therefore \quad \sum_{i=1}^{n} d_i/n \quad < \quad (d_0 + d_1)/2$$

$$\therefore \quad \bar{d} \quad < \quad (d_0 + d_1)/2$$

Note that, as indicated in Hiraoka and Kawashiwa (1993), the CBD distance of the core-ring boundary is $(d_1 + d_0)/2$ for the system of the continuous-linear region.

7) It is because, for the fixed d_0 and d_1 of a given railway-line region during a specific time-period, decentralization process of the spatial-cycle path would advance faster as the average CBD distance decreases through the spatial disaggregaion of localities in the vicinity of the CBD.

8) It is because, for the fixed d_0 and d_1, if the average CBD distance decreases through the spatial disaggregation of localities in the vicinity of the CBD, then the ROXY-index value tends to increase in case of decentralization stages.

9) As indicated by equation (9), through the multiplication of -1 to each value appearing in Tables 3 and 4, we can obtain its corresponding value of the ROXY-index for which the CBD distance would be used as its weighing factor. Note here that in the present case the reversed CBD distance is defined as "$d_1 + d_0 - x$."

10) As to the Tokaido-line region, the spatial-cycle paths diagrammatically illustrated by means of the circular-cyclic form for the disaggregated case show rather wild movements for both systems of the continuous-linear and fan-shaped regions. One of the major causes for this would be the existence of a relatively tall hill or a relatively deep hollow along the interpolated curve obtained for the annual growth ratio, between

the points of the CBD distance of 28.0km and 37.1km as can be observed in Figure A-2. The reason for the existence of such a drastic hill or hollow would be that Nishi-ku (CBD distance of 27.6km) is located extremely close (by distance of 0.4km) to Hodogaya-ku (CBD distance of 28.0km) as compared with the interval distance of 9.1km between Hodogaya-ku and its next-door outward locality, Totsuka-ku (CBD distance of 37.1km). In case that the annual growth ratios of Nishi-ku and Hodogaya-ku are significantly different, the possible steep positive slope between them causes a tall hill between Hodogaya-ku and Totsuka-ku, while the possible steep negative slope causes a deep hollow. These effects shall be referred to as the *overshooting effects*. In Figure A-2, the positive slope is making a hill (i.e., an overshooting-hill) for the periods 1960-65, 1970-75, 1975-80, 1980-85 and 1985-90, while the negative slope is making a hollow (i.e., an overshooting hollow) for the period 1965-70.

11) The value of $(d_i + d_o)/2$ is the CBD distance of the core-ring boundary for the system of the continuous-linear region as discussed in Hiraoka and Kawashima (1993). This boundary distance in the aggregated case and that in the disaggregated case are respectively 31.45km and 28.30km for the Chuo-line region, 32.70km and 31.10km for the Takasaki-line region, 27.70km and 26.10km for the Joban-line region, 28.75km and 26.10km for the Tokaido-line region, and 28.60km and 25.95km for the Sobu-line region. For each of the five railway-line regions, the difference between the above paired values is much smaller than that in the system of the discrete-linear region which we have discussed in the text. Therefore, in the system of the continuous-linear region, the spatial-cycle stage for the disaggregated case would be less advanced than that for the aggregated case as compared with the difference in the spatial-cycle stages between the aggregated and disaggregated cases observed in the system of the discrete-linear region. That is, the difference in the spatial-cycle stages between the aggregated and disaggregated cases is smaller for the system of the continuous-linear region than for the system of the discrete-linear region.

12) As indicated by equation (18), through the multiplication of −1 to each value appearing Tables 5 and 6, we can obtain its corresponding value of the ROXY-index for which the CBD distance would be used as its weighing factor. Note here that in the present case the reversed CBD distance is defined as "$d_i^2 + d_o^2 - x^2$."

13) The value of $\{(d_i^2 + d_o^2)/2\}^{\frac{1}{2}}$ indicates the CBD distance of the core-ring boundary for the system of the fan-shaped region as discussed in Hiraoka and Kawashima (1993). This boundary distance in the aggregated case and that in the disaggregated case are respectively 39.59km and 39.25km for the Chuo-line region, 41.34km and 41.12km for the Takasaki-line region, 34.34km and 34.07km for the Joban-line region, 35.81km and 35.46km for the Tokaido-line region, and 35.60km and 35.25km for the Sobu-line region. For each of the five railway-line regions, the difference between the above paired values is quite small as compared with the difference in the system of the discrete-linear region or the difference in the system of the continuous-linear region. Therefore, in the system

of the fan-shaped region, the spatial-cycle stage for the disaggregated case are theoretically supposed to be furtheremore less advanced than that for the aggregated case as compared with the difference in the spatial-cycle stage observed in both systems of the discrete-linear and continuous-linear regions as generally indicated in our empirical results shown by Figures 1 through 5.

References

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Appendix

Table A-1 shows the amended versions of equations (5.7) and (5.9) in Hiraoka and Kawashima (1993), including their derivation processes from equations (4.24) and (4.36) respectively. Table A-2 and Figure A-1, meanwhile, show correspondingly-amended tables and figures for Tables 22 and 23 and Figures 28 and 29 furnished by *op. cit.*

Table A-1   Amended Versions of Equations (5.7) and (5.9)

(1)   For equation (5.7)

$$R_{LN} = \left\{ \frac{2 \int_{d_0}^{d_1} x r_L^{t,t+1}(x)dx}{(d_1 + d_0) \cdot \int_{d_0}^{d_1} r_L^{t,t+1}(x)dx} - 1.0 \right\} \times \frac{d_1 + d_0}{d_1 - d_0} \times 10^4 \tag{4.24}$$

Two integrals in the above expression have the growth ratio $r_L^{t,t+1}(x)$ whose value can be obtained discretely over the distance x. We interpolate the discrete growth ratio through the following third-order spline function;

$$r_{LS}(x) = a_0 + a_1 x + \sum_{i=1}^{n} c_i (x - x_i)_+^3 \qquad (i = 1, 2, \cdots, n) \tag{5.6}$$

Then, the two integrals in equation (4.24) respectively turn out to be as follows;

$$
\begin{cases}
\int_{d_0}^{d_1} r_L^{t,t+1}(x)dx \; = \int_{d_0}^{d_1} r_{LS}(x)dx \\[2mm]
\qquad\qquad = a_0(x_n - x_1) + \frac{1}{2} a_1 \left(x_n^2 - x_1^2 \right) + \frac{1}{4} \sum_{i=1}^{n} c_i (x_n - x_i)^4 \\[4mm]
\int_{d_0}^{d_1} x r_L^{t,t+1}(x)dx \; = \int_{d_0}^{d_1} x r_{LS}(x)dx \\[2mm]
\qquad\qquad = \frac{1}{2} a_0 \left(x_n^2 - x_1^2 \right) + \frac{1}{3} a_1 \left(x_n^3 - x_1^3 \right) \\[2mm]
\qquad\qquad + \sum_{i=1}^{n} c_i \left\{ -\frac{1}{20} (x_n - x_i)^5 + \frac{1}{4} x_n (x_n - x_i)^4 \right\}
\end{cases}
\tag{5.7}
$$

Table A-1 (Continued)

(2) For equation (5.9)

$$R_{LN} = \left\{ \frac{2}{(d_1^2 + d_0^2)} \cdot \frac{\int_{d_0}^{d_1} r_F^{t,t+1}(x) \cdot x^3 dx}{\int_{d_0}^{d_1} r_F^{t,t+1}(x) \cdot x dx} - 1.0 \right\} \times \frac{d_1^2 + d_0^2}{d_1^2 - d_0^2} \times 10^4 \tag{4.36}$$

Two integrals in the above expression have the growth ratio $r_L^{t,t+1}(x)$ whose value can be obtained discretely over the distance x. We interpolate the discrete growth ratio through the following third-order spline function ;

$$r_{FS}(x) = a_0 + a_1 x + \sum_{i=1}^{n} c_i (x - x_i)_+^3 \tag{5.8}$$

Then, the two integrals in equation (4.36) respectively turn out to be as follows;

$$\left\{ \begin{array}{l}
\displaystyle \int_{d_0}^{d_1} x r_F^{t,t+1}(x) dx \ = \int_{d_0}^{d_1} x r_{FS}(x) dx \\[3mm]
\displaystyle \qquad\qquad = \frac{1}{2} a_0 \left(x_n^2 - x_1^2 \right) + \frac{1}{3} a_1 \left(x_n^3 - x_1^3 \right) + \sum_{i=1}^{n} c_i \left\{ -\frac{1}{20}(x_n - x_i)^5 + \frac{1}{4} x_n (x_n - x_i)^4 \right\} \\[3mm]
\displaystyle \int_{d_0}^{d_1} x^3 r_F^{t,t+1}(x) dx \ = \int_{d_0}^{d_1} x^3 r_{FS}(x) dx \\[3mm]
\displaystyle \qquad\qquad = \frac{1}{4} a_0 \left(x_n^4 - x_1^4 \right) + \frac{1}{5} a_1 \left(x_n^5 - x_1^5 \right) \\[3mm]
\displaystyle \qquad\qquad + \sum_{i=1}^{n} c_i \left\{ -\frac{1}{140}(x_n - x_i)^7 + \frac{1}{20} x_n (x_n - x_i)^6 - \frac{3}{20} x_n^2 (x_n - x_i)^5 + \frac{1}{4} x_n^3 (x_n - x_i)^4 \right\}
\end{array} \right.$$

$$\tag{5.9}$$

Table A-2 Amended Versions of Tables and 22 and 23
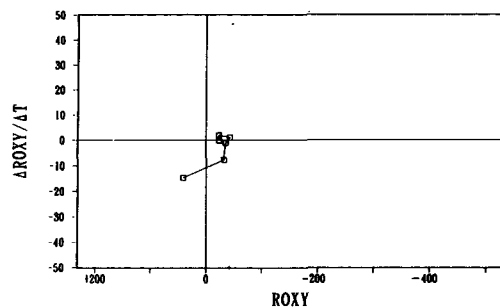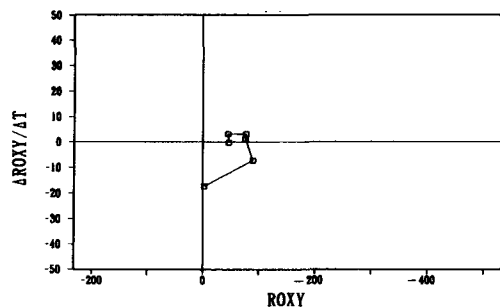
(1) For Table 22

Table 22 Values and Their Marginal Changes of the New Formulation
of ROXY Index for Continuous-linear Region

Period		1960–65	1965–70	1970–75	1975–80	1980–85	1985–90
Type-a	Value	40.80	−32.75	−35.40	−43.63	−23.73	−24.25
	Marginal change	−14.71	−7.62	−1.09	1.17	1.94	−0.10
Type-s	Value	−3.15	−90.10	−76.35	−77.51	−45.27	−46.54
	Marginal change	−17.39	−7.32	1.26	3.11	3.10	−0.25

(2) For Table 23

Table 23 Values and Their Marginal Changes of the New Formulation
of ROXY Index for Two-dimensional Fan-shaped Region

Period		1960–65	1965–70	1970–75	1975–80	1980–85	1985–90
Type-a	Value	59.53	58.97	18.42	−35.75	−15.83	−11.83
	Marginal change	−0.11	−4.11	−9.47	−3.43	2.39	0.80
Type-s	Value	66.24	33.44	−0.38	−53.59	−27.57	−23.48
	Marginal change	−6.56	−6.66	−8.70	−2.72	3.01	0.82

(a) For aggregated case

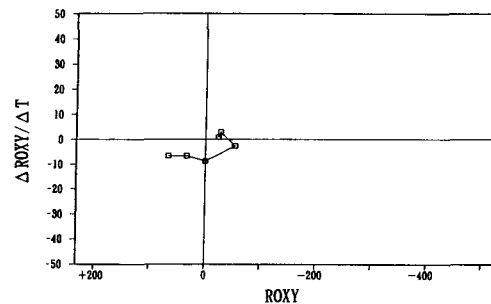(a) For aggregated case

(b) For disaggregated case

(b) For disaggregated case

Figure 28   New Formulation of ROXY Index in terms of
Continuous-linear Region: Values and Their Marginal
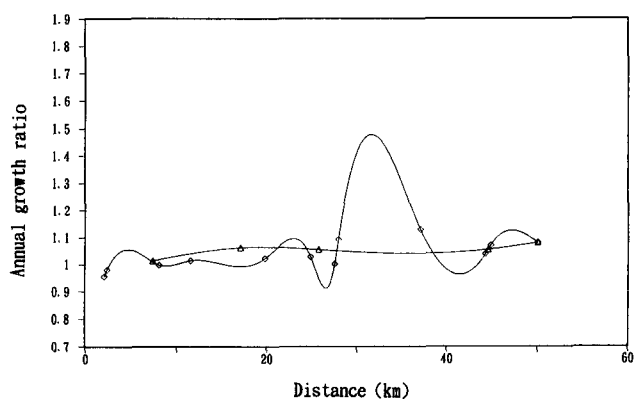Changes (Aggregated and Disaggregated cases) for
Takasaki-line Region

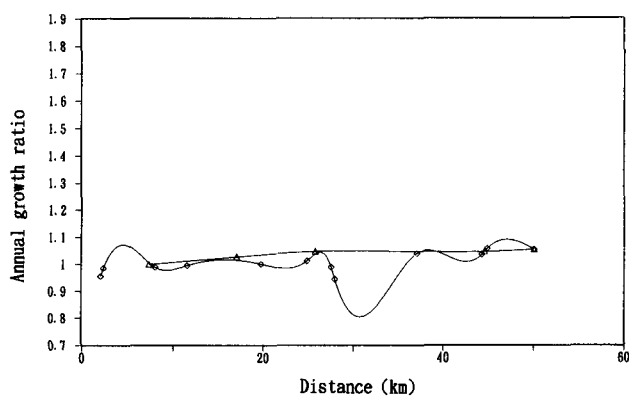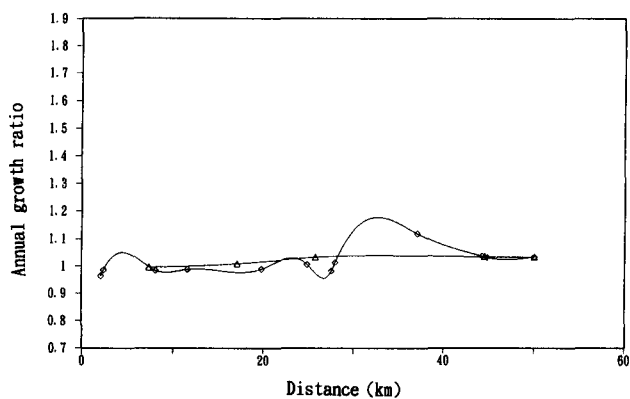(1)   For Figure 28

Figure 29   New Formulation of ROXY Index in terms of Two-
dimensional Fan-shaped Region: Values and Their
Marginal Changes (Aggregated and Disaggregated cases)
for Takasaki-line Region

(2)   For Figure 29

Figure A−1   Amended Versions of Figures 28 and 29
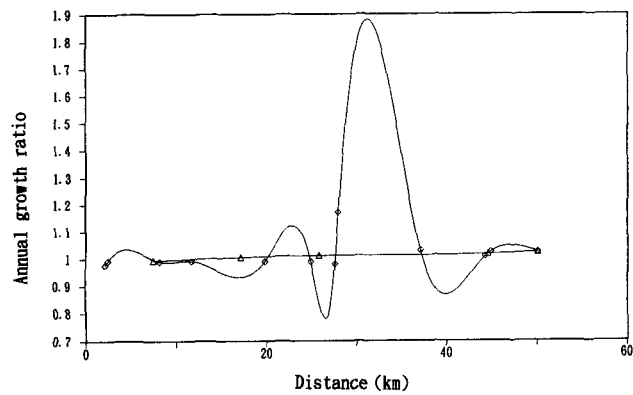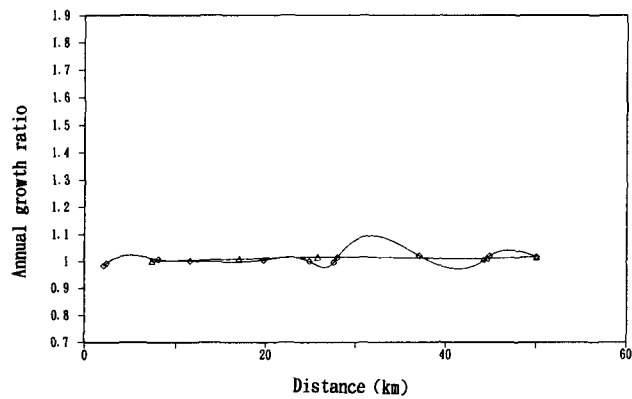
(a) 1960-65

(b) 1965-70

(c) 1970-75

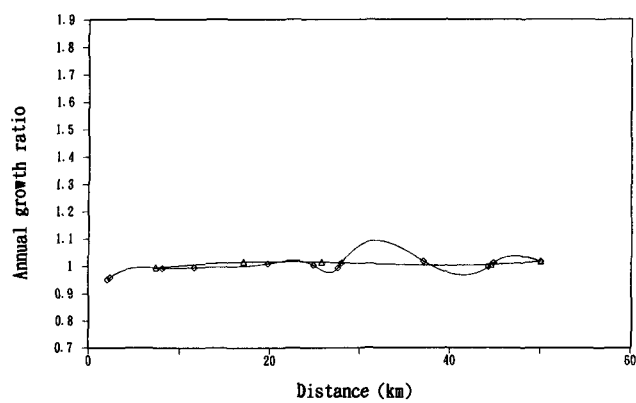*Notes*   ◇: disaggregated   △: aggregated

Figure A—2   Interpolated Annual Growth Ratio in Tokaido-line Region

(d) 1975-80

(e) 1980-85

(f) 1985-90

Notes ◇: disaggregated △: aggregated

Figure A−2 (Continued)

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